Can a Better Informed Listener be Easier to Persuade?*

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August 2020

Abstract

We study the impact of exogenous news on the classic Bayesian persuasion problem. The sender supplies information over multiple periods, but is unable to commit at the onset to the information that she will supply in periods ahead. A tension then emerges between the sender and her future self. We show that by resolving this tension, more informative news can make the sender better off.

JEL classification: C70, D72, D80

Keywords: Bayesian Persuasion, Dynamic Games, Information Provision

*The ideas in this paper originated from our collaboration with Jesper Rüdiger, whose contribution to our own understanding of the dynamic persuasion problem cannot be overestimated. We also thank Jeff Ely, Nenad Kos, Lucie Ménager, Konrad Mierendorff, Larry Samuelson, Johannes Schneider, Christoph Schottmüller and Peter Norman Sørensen for insightful conversations. Any remaining error is our own.

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1 Introduction

The by now classic Bayesian persuasion problem has found numerous applications, and therefore recently received a lot of attention. Yet one aspect of the problem that is still not well understood concerns the impact of exogenous news. The purpose of the present paper is to draw attention to the subtle effects of exogenous news on the Bayesian persuasion problem.

To illustrate the main idea of our paper, consider the example in Kamenica and Gentzkow (2011) of a lobbyist trying to convince a politician to take a certain action A. However, instead of modelling this situation as a static problem, imagine that we let the process unfold over multiple periods, thereby enabling the politician to accumulate information beyond the lobbyist's control. We think of this information as exogenous news, and ask how this affects the problem of the lobbyist.

Suppose that the lobbyist gets to meet the politician twice. In this setting, the politician can either choose an action after the first meeting, or wait until the second meeting in order to observe whatever news comes up in the time interval between the two meetings. The lobbyist is worse off in this setting as compared to the static setting, since to obtain action A after the first meeting the lobbyist must now additionally persuade the politician not to wait. In particular, in sharp contrast with the static setting, to obtain action A in the first period the lobbyist here supplies information that –from the politician's perspective– creates positive value.

Now suppose that the lobbyist gets to meet the politician thrice. The previous remarks concerning the value of information provided by the sender in the two-period setting imply that, with three periods, seen from period one, the lobbyist's current self is effectively playing against the lobbyist's future self. Indeed, with three periods, the reason for which the politician is tempted to wait may now comprise future information that he expects the lobbyist to supply.

The main contribution of our paper is to show that, in situations of the kind described above, increasing the informativeness of the news might end up making the sender better off. The mechanism operates as follows. Increasing the informativeness of the news sometimes reduces the amount of future information that the sender supplies. More informative news then weakens the aforementioned tension between the sender and her future self. We show that this indirect positive effect on the welfare of the sender may outweigh the direct negative effect resulting from the sender's loss of control on the information flow.[1]

[1] In contrast with our main result, Kolotilin (2015) shows that, in a static setting, the sender's payoff goes down with more public information.

Related Literature. The idea of a tension between the sender and her future self in the Bayesian persuasion problem appears in Au (2015), Basu (2018), Henry and Ottaviani (2019), and more recently in Che, Kim and Mierendorff (2020). Yet, the role of exogenous news is not considered in any of these papers. In Au (2015) and Basu (2018), the cutoff belief at which the receiver is indifferent between accepting and rejecting is private information of the receiver. In equilibrium, the sender engages in "intertemporal information discrimination", targeting more lenient types first. Yet future information generated for more stringent types raises lenient types' incentives to wait, thus raising the cutoff belief at which the latter can be persuaded to accept. Henry and Ottaviani and Che et al. introduce constraints on the information flow. In the former study, the tension between the sender and her future self leads both the cutoff belief at which the receiver accepts and the cutoff belief at which he rejects to be higher under no commitment than under commitment. Che et al. obtain a version of a folk theorem and establish the existence of an equilibrium in which the receiver's beliefs (concerning future information supplied) effectively force the sender to fully reveal the state.[2]

Our analysis builds on the model of Bizzotto, Rüdiger and Vigier (2020), with the important difference that each piece of news is inconclusive in the present paper. As explained in Section 4, the tension between the sender and her future self disappears when news is conclusive, either perfectly revealing the bad state, or perfectly revealing the good state. Our work is also related to Gentzkow and Kamenica (2017), Au and Kawai (2020) and Li and Norman (2020) who, instead of exogenous news, study persuasion with multiple senders. Orlov, Skrzypacz and Zryumov (2020) examine a sender that partly controls the information available to a receiver who faces a real option problem. Dynamic persuasion problems are also examined in Smolin (2018) and Ely and Szydlowski (2019), but in settings with no news, and in which the sender is able to dynamically commit to an information policy. Gratton, Holden and Kolotilin (2018) and Honryo (2018) examine related persuasion problems, but where the sender is privately informed, thus inducing a signalling problem.

The rest of the paper is organized as follows. The model is presented in Section 2. Section 3 contains the preliminary analysis. Our main result is presented in Section 4. Section 5 concludes.

[2]Brocas and Carrillo (2007) is connected, but the key tension in the aforementioned papers is absent in that model because the sender is restricted to making a single take-it-or-leave-it request to the receiver.

2 Model

There is a sender ("she") and a receiver ("he"). The state of the world ω is either H or L, with $\mathbb{P}(\omega = H) := p_1$. The receiver has three periods, indexed by t, to choose between two actions, called *accept* and *reject*. The receiver gets payoff 1 for taking the action that matches the state, i.e. accept in state H and reject in state L, and 0 otherwise. Waiting will allow the receiver to gather information about the state, in a way to be specified shortly. The sender gets payoff 1 if the receiver accepts and 0 otherwise. Both players discount time at rate $\delta \in (0, 1)$. All information being public, the players share common beliefs about the state. The model described in this section is the most parsimonious model of this kind permitting us to illustrate the main insight of our paper. Extensions are discussed in Section 4.

The (evolving) probability assigned to $\omega = H$ will be referred to as the belief. We allow pieces of news to be observed twice in the course of the game: once between periods 1 and 2, and once between periods 2 and 3. Each piece of news is an independent draw from the conditional probability distribution $\pi(\cdot \,|\, \omega)$ over $\{h, \ell\}$, where

$$\pi(h \,|\, H) = \pi(\ell \,|\, L) = \frac{1+\gamma}{2}, \qquad \gamma \in [0, 1].$$

Perfectly uninformative news corresponds to $\gamma = 0$, and perfectly informative news to $\gamma = 1$. Furthermore, an increase in γ increases the informativeness of the news in the sense of Blackwell (Blackwell (1953)).

The sender chooses in every period what additional information to generate. We model this choice as a splitting $\tau_t(p_t) \in \Delta([0, 1])$ (Aumann et al. (1995)) of the beginning-of-period-t belief p_t, and let q_t denote the resulting posterior belief. When $\tau_t(x)$ is the degenerate distribution assigning probability 1 at x we say that (given $p_t = x$) the sender supplies no information (in period t). Importantly, the sender is unable to commit in period 1 to the information she will supply in periods ahead.

The timeline is as follows. The game starts at $t = 1$ and ends whenever the receiver acts (i.e. either accepts or rejects). The sender first chooses a splitting of p_1, that induces the end-of-period-1 belief q_1. The receiver then chooses between accept, reject and wait. If the receiver acts, payoffs are realized. Otherwise the first piece of news, s_1, is observed, inducing the beginning-of-period-2 belief p_2. The previous sequence repeats in period 2: the sender chooses a splitting of p_2, and the receiver chooses between accept, reject and wait. If he waits, the second piece of news, s_2, is observed, inducing the beginning-of-period-3 belief p_3. The sender then chooses a splitting of p_3, and the receiver acts.

The equilibrium concept is Perfect Bayesian Equilibrium (PBE): the player at each decision node maximizes her/his expected payoff conditional on (a) the other player's strategy and (b) the belief obtained using Bayes' rule. We focus on PBE such that: (i) whenever the sender is indifferent between two splittings ordered by Blackwell's criterion, she chooses the least informative of the two; (ii) whenever indifferent between two decisions, the receiver makes the decision preferred by the sender. These refinements simplify the exposition, but are inessential for our results. PBE satisfying (i) and (ii) will be referred to as equilibria for short. The existence of a unique equilibrium is established in the Appendix.

Our dynamic persuasion model is for $\delta(1+\gamma)\leq 1$ equivalent in practice to the static persuasion model of Kamenica and Gentzkow (2011).[3] To focus on the interesting case, we assume in the rest of the paper that $\delta(1+\gamma)>1$.

3 Preliminaries

We present in this section the main features of the equilibrium. All results in this section are proven in the Appendix.

At $t=3$ the receiver must choose one of the two actions. The receiver accepts if $q_3\geq 1/2$ and rejects otherwise. However, in earlier periods the receiver may choose to wait in order to accumulate information about the state. Let $g_t^e(q_t)$ denote the receiver's equilibrium expected payoff evaluated at the end of period t. Then, for $t=1,2$:[4]

$$g_t^e(q_t)=\max\left\{1-q_t,\delta\mathbb{E}_{s_t,\tau_{t+1}^e}[g_{t+1}^e(q_{t+1})\,|\,q_t],q_t\right\}. \tag{1}$$

In particular, if $\delta\mathbb{E}_{s_t,\tau_{t+1}^e}[g_{t+1}^e(q_{t+1})\,|\,q_t]>\max\{1-q_t,q_t\}$ then at the end of period t the receiver chooses to wait. As we next show, this occurs if and only if the corresponding belief lies in an interval around 1/2.

Lemma 1. *There exist cutoffs $a_1^e\leq a_2^e<a_3^e=1/2=b_3^e<b_2^e\leq b_1^e$ such that in equilibrium the receiver rejects if $q_t<a_t^e$, waits if $q_t\in[a_t^e,b_t^e)$, and accepts if $q_t\geq b_t^e$.*

The lower b_t^e the more type II errors (namely, accepting when $\omega=L$) the sender can induce the receiver to make. As Lemma 1 shows that $b_1^e\geq b_2^e>b_3^e$, this suggests that the sender may

[3] In particular, in this case, in equilibrium the receiver never waits: the receiver rejects for $q_1<1/2$ and accepts for $q_1\geq 1/2$.

[4] Where τ_t^e denotes the equilibrium splitting of the sender in period t.

prefer to postpone the time at which she will try to persuade the receiver to accept, and, in order to do so, may find it optimal to supply little or no information at $t=1$. Our two next lemmata summarize the main features of the sender's equilibrium strategy.

Lemma 2. *In equilibrium, for* $p_t \geq b_t^e$ *the sender supplies no information. For* $p_t \in (0, b_t^e)$*, either (i) the sender splits every* $p_t \in (0, b_t^e)$ *on 0 and* b_t^e*, or (ii) at every* $p_t \in (0, b_t^e)$ *the sender generates strictly less information, in the sense of Blackwell, than in the former case.*

Henceforth, say that the sender is aggressive in period t if case (i) of the lemma holds, and that she is conservative in period t if case (ii) holds. The following specifics of the sender's equilibrium strategy will be useful in the rest of the analysis.[5]

Lemma 3. *In equilibrium, the sender is aggressive in period 3. When the sender is conservative in period 2:*

- *for* $p_2 \in (0, a_2^e)$ *the sender splits* p_2 *on 0 and* a_2^e*;*
- *for* $p_2 = a_2^e$ *the sender supplies no information in period 2;*
- *for* $p_2 \in (a_2^e, b_2^e)$ *the sender splits* p_2 *on* a_2^e *and* b_2^e*.*

The relevant parametric regions in regard to the sender's equilibrium strategy are depicted in Figure 1: in equilibrium, the sender is conservative in period 1 everywhere in gray, and aggressive elsewhere; in period 2, the sender is aggressive below the dashed curve, and conservative above it. We close the section by providing some intuition for the main features of the figure.[6] First, notice that for δ close to 1 in equilibrium the sender is conservative in periods 1 and 2. The logic is simple: increasing δ not only reduces the sender's impatience, but also makes it harder to convince the receiver not to wait for information. So increasing δ unambiguously raises the sender's incentive to postpone persuasion and, hence, to be conservative. The impact of γ is more complicated. On one hand, increasing γ raises the receiver's incentive to wait for news, thereby making it harder for the sender to persuade the receiver to accept before period 3. On the other hand, raising γ reduces the scope for manipulating beliefs once news has been observed. This, in turn, incentivizes the sender to try persuading the receiver before news is observed. For instance, if γ is close to 1, then clearly the sender can do no better than to be aggressive in period t, for $t=1,2$. The upshot is a non-monotonic

[5]The details of the information that in equilibrium the sender supplies when she is conservative in period 1 are unimportant. A complete characterization is provided in the Appendix.

[6]See Bizzotto *et al.* (2020) for a complementary discussion.

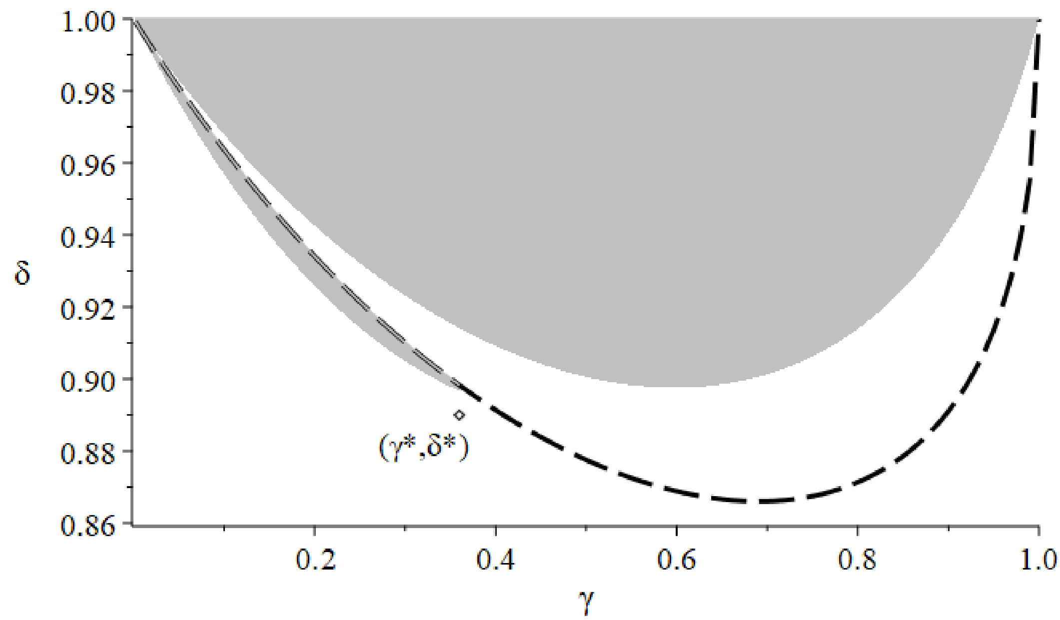

FIGURE 1

effect of γ on the sender's equilibrium strategy: each period $t = 1, 2$ the sender is aggressive if γ is close to either 0 or 1, but conservative in an intermediate interval.[7]

4 Main Result

We present our main result in Subsection 4.1. A discussion follows in Subsection 4.2.

4.1 Analysis

In the first part of this subsection we show that whether the sender is aggressive in period 2 determines whether future information generated by the sender is valuable for the receiver. The second part of the subsection draws the former result's implications, and shows that increasing the informativeness of the news can make the sender better off.

Extending previous notation, let $g_t^{\emptyset}(q_t)$ denote the receiver's expected payoff evaluated at

[7]Another noticeable feature of the figure is that the gray region above the dashed curve is strictly smaller than the area above said curve, indicating a sense in which the sender tends be more aggressive in period 1 than in period 2. This in turn follows from the way the cutoff b_t^e evolves over time: when, as is the case here, the difference $b_2^e - b_3^e$ is large relative to $b_1^e - b_2^e$ the sender has stronger incentives to postpone persuasion in period 2 than in period 1.

the end of period t, but, this time, in the single-player setting in which the sender never supplies any information.[8] As in (1),

$$g_t^{\emptyset}(q_t) = \max\left\{1 - q_t, \delta \mathbb{E}_{s_t}[g_{t+1}^{\emptyset}(q_{t+1}) \,|\, q_t], q_t\right\}.$$

Moreover, one proves as in Lemma 1 the existence of cutoffs $a_1^{\emptyset} \leq a_2^{\emptyset} < a_3^{\emptyset} = 1/2 = b_3^{\emptyset} < b_2^{\emptyset} \leq b_1^{\emptyset}$ such that, in the single-player setting, the receiver rejects if $q_t < a_t^{\emptyset}$, waits if $q_t \in [a_t^{\emptyset}, b_t^{\emptyset})$, and accepts if $q_t \geq b_t^{\emptyset}$.[9] Notice that any information supplied by the sender evidently makes waiting (weakly) more attractive for the receiver, hence $a_1^e \leq a_1^{\emptyset}$ and $b_1^e \geq b_1^{\emptyset}$. We will say that future information generated by the sender is valuable for the receiver if $a_1^e < a_1^{\emptyset}$ and $b_1^e > b_1^{\emptyset}$, and that future information generated by the sender has no value for the receiver if $a_1^e = a_1^{\emptyset}$ and $b_1^e = b_1^{\emptyset}$. We occasionally write $p_t(\tilde{q}, h)$ for the realization of p_t conditional on $q_{t-1} = \tilde{q}$ and $s_{t-1} = h$; the belief $p_t(\tilde{q}, \ell)$ is similarly defined. Note that $p_t(\tilde{q}, h) > \tilde{q} > p_t(\tilde{q}, \ell)$ for all $\tilde{q} \in (0,1)$, since $\delta(1+\gamma) > 1$.

Proposition 1. *Let $\gamma < 1$. In equilibrium, future information generated by the sender is valuable for the receiver if and only if the sender is aggressive in period 2.*

Proof: Notice to begin with that, by Lemmata 2 and 3, in equilibrium the sender splits any $p_3 \in (0, 1/2)$ on 0 and 1/2, and supplies no information if $p_3 \geq 1/2$. Thus:

$$g_2^e(q_2) = g_2^{\emptyset}(q_2) = \max\{1 - q_2, \delta(1+\gamma)/2, q_2\} \tag{2}$$

and, in particular, $[a_2^e, b_2^e] = [a_2^{\emptyset}, b_2^{\emptyset}]$.

Suppose that in equilibrium the sender is conservative in period 2. We will show that in this case future information generated by the sender has no value for the receiver. By (2), g_2^e is affine both on $[0, a_2^e]$ and on $[a_2^e, b_2^e]$. Hence, by the second part of Lemma 3:

$$\delta \mathbb{E}_{s_1, \tau_2^e}[g_2^e(q_2) \,|\, q_1] = \delta \mathbb{E}_{s_1}[g_2^e(q_2) \,|\, q_1].$$

And since $g_2^e = g_2^{\emptyset}$:

$$\delta \mathbb{E}_{s_1}[g_2^e(q_2) \,|\, q_1] = \delta \mathbb{E}_{s_1}[g_2^{\emptyset}(q_2) \,|\, q_1].$$

Hence,

$$\delta \mathbb{E}_{s_1, \tau_2^e}[g_2^e(q_2) \,|\, q_1] = \delta \mathbb{E}_{s_1}[g_2^{\emptyset}(q_2) \,|\, q_1].$$

[8] As in Wald (1947).

[9] This follows from Proposition 3 in the Appendix.

The left-hand side of this equation is, in equilibrium, the receiver's expected payoff from waiting at the end of period 1; the right-hand side of this equation is the corresponding payoff in the single-player setting. The two being equal, we obtain $a_1^e = a_1^\emptyset$ and $b_1^e = b_1^\emptyset$.

Next, suppose that in equilibrium the sender is aggressive in period 2. We will show that in this case future information generated by the sender is valuable for the receiver. The receiver's expected payoff from waiting at the end of period 1 can now be written as

$$\delta \mathbb{E}_{s_1, \tau_2^e}[g_2^e(q_2) \,|\, q_1] = \delta \mathbb{E}_{s_1}[k(q_2) \,|\, q_1],$$

where k is the piecewise affine continuous function with a single kink at $b_2^e = b_2^\emptyset$ satisfying $k(0) = 1$, $k(b_2^\emptyset) = b_2^\emptyset$ and $k(1) = 1$. In particular, $k(q_2) = g_2^\emptyset(q_2)$ for all $q_2 \in \{0\} \cup [b_2^\emptyset, 1]$ and $k(q_2) > g_2^\emptyset(q_2)$ for all $q_2 \in (0, b_2^\emptyset)$. Now, given $\gamma < 1$, observe that $p_2(q_1, \ell) \in (0, b_2^\emptyset)$ for any $q_1 \in (0, b_1^\emptyset]$.[10] We conclude that, for $q_1 \in (0, b_1^\emptyset]$:

$$\delta \mathbb{E}_{s_1}[k(q_2) \,|\, q_1] > \delta \mathbb{E}_{s_1}[g_2^\emptyset(q_2) \,|\, q_1].$$

This, in turn, yields $a_1^e < a_t^\emptyset$ and $b_1^e > b_1^\emptyset$.

■

We illustrate Proposition 1 in Figure 2, for $\delta = .89$. In this case, in equilibrium the sender is conservative in period 2 for $\gamma \in [.41, .90]$ and aggressive otherwise. The horizontal axis measures the informativeness of the news, γ. The dashed black curve depicts the graph of the equilibrium cutoff b_1^e as a function of γ, and the dashed gray curve the cutoff $b_1^\emptyset$ obtained when the news is the receiver's only information source. The solid curves similarly represent a_1^e and $a_1^\emptyset$. The black and gray curves coincide in the interval $[.41, .90]$. Everywhere else, the dashed black curve lies above the dashed gray curve, while the solid black curve lies below the solid gray curve. We are now ready to state the paper's main result.

Proposition 2. *Increasing the informativeness γ of the news can increase the sender's equilibrium expected payoff.*

Proof: Let $\overline{b}_1$ denote the receiver's period-1 acceptance cutoff assuming the sender is aggressive in period 2. Then $\overline{b}_1$ is evidently continuous in γ, since $b_2^e = b_2^\emptyset = \delta(1+\gamma)/2$. Moreover, the arguments in the proof of Proposition 1 establish that $\overline{b}_1 > b_1^\emptyset$ as long as $\gamma < 1$. Next,

[10]Otherwise we could find q_1 such that, in the single-player setting, the receiver chooses to wait at the end of period 1 knowing that he will accept with probability 1 in period 2.

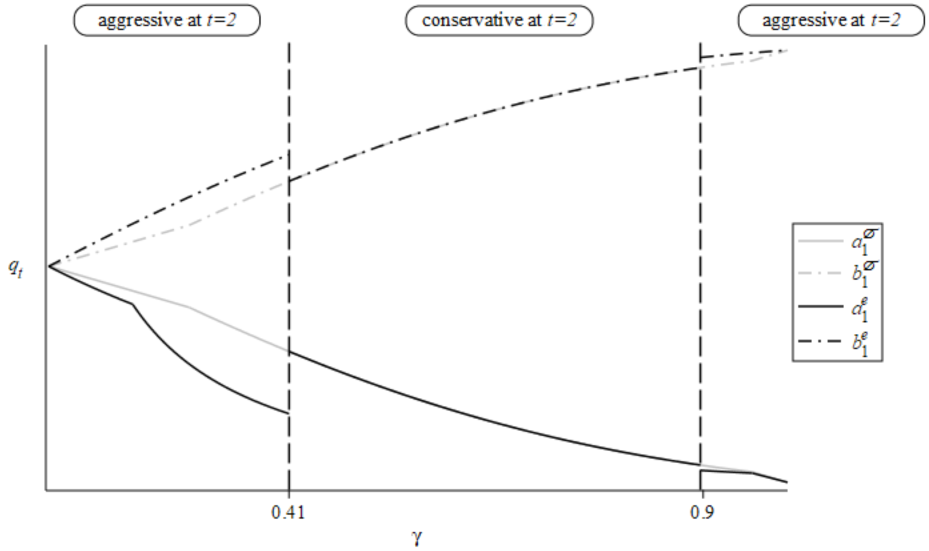

FIGURE 2

let the equation $\delta=\varphi_2(\gamma)$ represent the parametric frontier separating the regions where in equilibrium the sender is, respectively, aggressive in period 2 and conservative in period 2. The function φ_2 is decreasing in an open interval $(\underline{\gamma},\overline{\gamma})$ such that in equilibrium the sender is aggressive in period 2 immediately to the left of the graph of φ_2, and conservative in period 2 immediately to the right of said graph (see the Appendix for an analytic proof, and Figure 1 for an illustration). Pick $\tilde{\gamma}\in(\underline{\gamma},\overline{\gamma})$ and fix $\delta=\varphi_2(\tilde{\gamma}):=\tilde{\delta}$. Since $\overline{b}_1(\tilde{\gamma},\tilde{\delta})>b_1^{\emptyset}(\tilde{\gamma},\tilde{\delta})$, the continuity of $\overline{b}_1$ gives $r>0$ such that

$$\overline{b}_1(\dot{\gamma},\tilde{\delta})>b_1^{\emptyset}(\ddot{\gamma},\tilde{\delta}),\qquad \text{for all } \dot{\gamma},\ddot{\gamma}\in B(\tilde{\gamma},r).$$

Now choose $\dot{\gamma}<\tilde{\gamma}<\ddot{\gamma}$ with $\dot{\gamma},\ddot{\gamma}\in B(\tilde{\gamma},r)$. Then $b_1^e(\dot{\gamma},\tilde{\delta})=\overline{b}_1(\dot{\gamma},\tilde{\delta})$ and, by Proposition 1, $b_1^e(\ddot{\gamma},\tilde{\delta})=b_1^{\emptyset}(\ddot{\gamma},\tilde{\delta})$. This shows that for any $p_1\in\left[b_1^{\emptyset}(\ddot{\gamma},\tilde{\delta}),\overline{b}_1(\dot{\gamma},\tilde{\delta})\right)$ the sender's equilibrium expected payoff is equal to 1 if $\gamma=\ddot{\gamma}$ and is strictly less than 1 if $\gamma=\dot{\gamma}$.[11]

■

The basic mechanism behind Proposition 2 is as follows. To each parameter pair (γ,δ) is associated one of two equilibrium regimes. In regime I (viz. when the sender is aggressive in period 2), future information supplied by the sender creates positive value for the receiver

[11]For more general conditions under which increasing γ increases the sender's equilibrium expected payoff see the Online Appendix, Proposition 4.

at $t=1$. By contrast, in regime II (viz. when the sender is conservative in period 2), future information supplied by the sender creates no value for the receiver at $t=1$. To the extent that it simplifies the task of persuading the receiver to accept at $t=1$, regime II is more favorable to the sender than regime I. Yet, increasing γ can induce the sender to switch from regime I to regime II. As regime II is more favorable to the sender than regime I, increasing γ ultimately benefits the sender in period 1. In Figure 1 for example, at $(\gamma,\delta)=(\gamma^*,\delta^*)$, increasing the informativeness of the news by as much as 20% still enables the sender to increase the equilibrium probability with which she can persuade the receiver to accept.

4.2 Discussion

Scope. Many of the assumptions of the model can be relaxed without affecting the basic mechanism described in the last paragraph of the previous subsection. Augmenting the number of periods, for instance, would complicate the equilibrium characterization, but would not affect our main result. Neither would an asymmetric news structure such that, say, $\pi(\ell \,|\, L)=\gamma$ whereas $\pi(h \,|\, H)=\gamma+\varepsilon$. The symmetric nature of the receiver's payoffs saves on notation but is of course inessential. The sender and the receiver could have different discount rates; in fact, our main result would continue to hold even if the sender did not discount payoffs. On the other hand, the mechanism above ceases to work in a two-period variant of the model (that is, when a single piece of news is observed). Nor does it operate with conclusive news: in the perfect good news case, namely, when $\pi(\ell \,|\, L)=1$ and $\pi(h \,|\, H)=\gamma$, regime I prevails regardless of the parameters, thus obstructing the necessary regime switch; with perfect bad news, namely, when $\pi(\ell \,|\, L)=\gamma$ and $\pi(h \,|\, H)=1$, the regime switch does occur but the mechanism breaks down because regime II ceases to be more favorable to the sender than regime I. Indeed, one shows that in this case $b_1^e=b_2^e=b_1^\emptyset=b_2^\emptyset$ irrespective of whether in period 2 the sender is aggressive or conservative.[12] Intuitively, with perfect bad news, future information supplied by the sender never affects the acceptance cutoff due to the fact that $s_t=\ell$ sends the belief to 0, at which point the receiver already knows the state.

The sender's commitment problem. The driver of our main result is the tension existing between the sender and her future self, in other words, the commitment problem faced by the sender. To illustrate this point, consider $(\gamma,\delta)=(\gamma^*,\delta^*)$ in Figure 1 and $p_1<b_1^\emptyset$. In this case,

[12] See Bizzotto *et al.* (2020).

the sender is aggressive in period 1 and 2. Her equilibrium expected payoff is thus $p_1/\bar{b}_1$.[13] Suppose now that the sender were able to commit in period 1 to information supplied in periods ahead. The sender could then split p_1 on 0 and $b_1^{\emptyset}$ and commit not to supply any information in subsequent periods. Knowing that only the news will be observed, the receiver accepts at $q_1 = b_1^{\emptyset}$. The sender's expected payoff is thus $p_1/b_1^{\emptyset}$. As $b_1^{\emptyset} < \bar{b}_1$ the previous arguments establish that, as long as in equilibrium the sender is aggressive in all periods, the sender would be strictly better off if she could commit to information supplied in periods ahead. By contrast one shows that whenever in equilibrium the sender is conservative in period 2 then the sender gains nothing from the ability to commit. Hence Proposition 2 can be interpreted as showing that increasing the informativeness of the news can in certain circumstances resolve the sender's commitment problem.

The receiver's incentive to acquire public information. An immediate corollary of our analysis is that increasing the informativeness of the news can lower the receiver's equilibrium expected payoff. To see this, notice that whenever the sender is aggressive in period 1 the receiver's expected payoff can be written as[14]

$$\left(\frac{p_1}{b_1^e}\right)b_1^e + 1 - \frac{p_1}{b_1^e} = 1 + p_1\left(1 - \frac{1}{b_1^e}\right).$$

So an increase in b_1^e implies an increase of the receiver's expected payoff. Now fix $\delta = .89$. As illustrated in Figure 2, for $\gamma < 0.41$, information supplied by the sender at $t = 2$ creates value for the receiver and raises b_1^e above $b_1^{\emptyset}$. A regime switch occurs at $\gamma = .41$, where b_1^e is brought down to $b_1^{\emptyset}$. Hence, by the previous remarks, the receiver's equilibrium expected payoff falls at $\gamma = .41$. This example shows that in certain cases, even if public information were free, the receiver might prefer forgoing such information.

Partial sender commitment. Suppose that the sender can commit to some long-term information policy, but she cannot commit not to further supply information on top of this policy. Then an immediate corollary of our analysis is that in certain circumstances the sender would commit to supply a minimum amount of information.[15] At $(\gamma, \delta) = (\gamma^*, \delta^*)$ in Figure 1 for example, the sender would benefit from committing to supply a small amount of

[13] We use here notation from the proof of Proposition 2, where $\bar{b}_1$ denotes the receiver's period-1 acceptance cutoff assuming the sender is aggressive in period 2.

[14] We are implicitly assuming $p_1 < b_1^e$.

[15] We thank an anonymous referee for suggesting this.

information in period 3. Doing this raises b_2^e slightly, thereby assuring the dynamic consistency of being conservative in period 2. This, in turn, benefits the sender by lowering the period-1 acceptance cutoff b_1^e.

5 Conclusion

We study a dynamic version of the canonical persuasion problem in which the sender supplies information over multiple periods, but is unable to commit in period 1 to the information she will supply in periods ahead. In the absence of additional sources of information, this problem reduces to the canonical (static) problem, in which case the sender gains nothing from the ability to commit. However, in the presence of exogenous news, a commitment problem emerges: future information that the sender supplies may then increase the period-1 cutoff belief at which the receiver can be persuaded to accept. When this occurs, the sender would be better off if she could commit not to supply future information. Our main insight is to show that in this case increasing the informativeness of the news can resolve the sender's commitment problem and even make her better off. The reason is that increasing the informativeness of the news can reduce the amount of future information that the sender supplies, thus easing the persuasion problem she faces in period 1.

Appendix

In this appendix we fully characterize the equilibrium, and demonstrate in the process that this equilibrium exists and is unique. The equilibrium characterization serves as a proof for all lemmas in Section 3.

To shorten notation, we define $\eta := (1+\gamma)/2$ and will use $M_t(p_t)$ as a shorthand for the support of $\tau_t(p_t)$. Using Bayes' rule, notice that $M_t(p_t)$ uniquely determines $\tau_t(p_t)$ whenever $|M_t(p_t)| \leq 2$; we will repeatedly make use of this remark in this appendix. The rest of the notation is as defined in the body of the paper. In particular, $g_t^e(q_t)$ (respectively, $f_t^e(q_t)$) denotes the receiver's (resp., the sender's) equilibrium expected payoff at q_t. The following result (see Bizzotto *et al.* (2020) for a proof) assures that $g_t^e(q_t)$ is a convex function of q_t for every t.

Proposition 3. *If $\phi : [0,1] \to \mathbb{R}$ is convex (respectively concave) then $\mathbb{E}_{s_t}[\phi(p_{t+1})|q_t]$ is convex (resp. concave) in q_t.*

Combining (1) and Proposition 3 establishes that $g_t^e(q_t)$ is the upper envelope of three convex functions representing respectively the receiver's expected payoff from rejecting, waiting and accepting. The cutoffs a_t^e and b_t^e are therefore well defined; by the same reasoning, so are the single-player-setting cutoffs $a_t^\emptyset$ and $b_t^\emptyset$.[16] In the remainder of this appendix, we construct the equilibrium by backward induction.

Period 3. In equilibrium, the receiver accepts if $q_3 \geq 1/2$ and rejects otherwise, so, in the notation of Lemma 1, $a_3^e = b_3^e = 1/2$. A standard concavification argument now shows that, in equilibrium, $M_3(p_3) = \{0, 1/2\}$ for $p_3 \in (0, 1/2)$ and $M_3(p_3) = \{p_3\}$ otherwise.

Period 2. As stated in (2), $g_2^e(q_2) = g_2^\emptyset(q_2) = \max\{1-q_2, \delta\eta, q_2\}$ and, therefore, $a_2^e = 1-\delta\eta$ and $b_2^e = \delta\eta$ (recall $\delta\eta > 1/2$, and so $b_2^e > 1/2 > a_2^e$). Next,

$$f_2^e(q_2) = \begin{cases} 0 & \text{if } q_2 < a_2^e, \\ \delta\left[\mathbb{P}(s_2 = \ell|q_2)2p_3(q_2,\ell) + \mathbb{P}(s_2 = h|q_2)\right] & \text{if } q_2 \in [a_2^e, b_2^e), \\ 1 & \text{if } q_2 \geq b_2^e. \end{cases}$$

[16] See Subsection 4.1 for all definitions pertaining to the single-player setting.

Notice that f_2^e is affine on the interval $[a_2^e, b_2^e)$. Concavifying f_2^e immediately shows that in equilibrium either $M_2(p_2)=\{0,b_2^e\}$ for all $p_2\in(0,b_2^e)$ (namely, the sender is aggressive in period 2), or else $M_2(p_2)=\{0,a_2^e\}$ for $p_2\in(0,a_2^e)$, $M_2(a_2^e)=\{a_2^e\}$, and $M_2(p_2)=\{a_2^e,b_2^e\}$ for $p_2\in(a_2^e,b_2^e)$ (namely, the sender is conservative in period 2), as recorded in Lemma 3.[17] In particular, in equilibrium the sender is conservative in period 2 if and only if, for $p_2=a_2^e$, generating no information yields an expected payoff at least as large as splitting a_2^e on 0 and b_2^e. This condition is equivalent to:

$$\frac{1-\delta\eta}{\delta\eta}\le f_2^e(a_2^e)=\delta(2-\eta-\eta\delta). \tag{3}$$

A couple of remarks will be useful for future reference. First, note that $g_2^e=g_2^\emptyset$ is affine on each of the intervals $[0,a_2^e]$ and $[a_2^e,b_2^e]$. Therefore $g_1^e=g_1^\emptyset$ whenever in equilibrium the sender is conservative in period 2. Second, we can rewrite condition (3) as $\delta\ge\varphi_2(\eta)$, where φ_2 is a continuous function over the interval $[1/2,1]$, $\varphi_2(1/2)=\varphi_2(1)=1$, and $\varphi_2(\eta)<1$ for $\eta\in(1/2,1)$. The last property can be immediately verified: for $\delta=1$ and $\eta\in(1/2,1)$ the cutoff a_2^e satisfies $p(a_2^e,h)=1/2$, which implies that for $p_2=a_2^e$ by supplying no information in period 2 the sender is able to obtain expected payoff a_2^e/b_3^e; this is greater than the expected payoff a_2^e/b_2^e obtained by splitting a_2^e on 0 and b_2^e (since $b_2^e>b_3^e$ whenever $\delta\eta>1/2$). In particular, it follows that $\varphi_2(\cdot)$ decreases on a non-empty open interval. The graph of the function φ_2 is depicted by the dashed curve in Figure 1.

Period 1. We consider separately the cases in which in equilibrium the sender is, respectively, conservative in period 2 (i.e. $\delta\ge\varphi_2(\eta)$) and aggressive in period 2 (i.e. $\delta<\varphi_2(\eta)$).

Case A: $\delta\ge\varphi_2(\eta)$. Recall to begin with that in this case $g_1^e=g_1^\emptyset$. In particular, $a_1^e=a_1^\emptyset$ and $b_1^e=b_1^\emptyset$. Let $\underline{c}_1$ and $\overline{c}_1$ be respectively implicitly defined by $p_2(\underline{c}_1,h)=b_2^e$, and $p_2(\overline{c}_1,\ell)=a_2^e$.[18] One easily checks that $\underline{c}_1<1/2<\overline{c}_1$. Moreover, one shows that $a_1^\emptyset<a_2^\emptyset$ whenever in equilibrium the sender is conservative in period 2.[19]

We claim that $a_1^\emptyset<\underline{c}_1$. Suppose by way of contradiction that the claim is false. Then $p_2(a_1^\emptyset,h)\ge b_2^e$ which, in turn, implies $a_1^\emptyset=a_2^\emptyset$. Yet we saw earlier that $a_1^\emptyset<a_2^\emptyset$, so the claim

[17] Trivially, $M_2=\{p_2\}$ for all $p_2\ge b_2^e$, regardless of whether the sender is aggressive or conservative.

[18] This gives $\underline{c}_1=\frac{\delta(1-\eta)}{1+\delta-2\delta\eta}=1-\overline{c}_1$.

[19] Consider (η,δ) such that $a_1^\emptyset\ge a_2^\emptyset$. We will show that in equilibrium the sender is then aggressive in period 2. Note that $a_1^\emptyset=a_2^\emptyset$ if and only if for $p_2(a_2^\emptyset,h)\ge b_2^\emptyset$, that is, if and only if

$$\frac{(1-\delta\eta)\eta}{(1-\delta\eta)\eta+\delta\eta(1-\eta)}\ge\delta\eta,$$

must be true. A similar argument implies $\overline{c}_1 < b_1^{\emptyset}$. In sum:

$$a_1^e = a_1^{\emptyset} < \underline{c}_1 < 1/2 < \overline{c}_1 < b_1^{\emptyset} = b_1^e.$$

Next, we calculate a_1^e and b_1^e. At $p_1 = a_1^e$, in equilibrium the receiver is indifferent between rejecting and waiting, giving

$$1 - a_1^e = \delta\left[\mathbb{P}(s_1 = \ell | a_1^e)\big(1 - p_2(a_1^e, \ell)\big) + \mathbb{P}(s_1 = h | a_1^e)\delta\eta\right]$$

and ultimately

$$a_1^e = \frac{1 - \delta\eta - \delta^2\eta(1-\eta)}{1 - \delta\eta + \delta^2\eta(2\eta - 1)}.$$

As $g_1^e = g_1^{\emptyset}$, we immediately get $b_1^e = 1 - a_1^e$.

Now define

$$\psi_I(q_1) := \delta\left[\mathbb{P}(s_2 = \ell \,|\, q_1)\frac{p_2(q_1,\ell)}{a_2^e} f_2^e(a_2^e) + \mathbb{P}(s_2 = h \,|\, q_1)\left(\frac{b_2^e - p_2(q_1,h)}{b_2^e - a_2^e} f_2^e(a_2^e) + \frac{p_2(q_1,h) - a_2^e}{b_2^e - a_2^e}\right)\right],$$

$$\psi_{II}(q_1) := \delta\left[\mathbb{P}(s_2 = \ell \,|\, q_1)\frac{p_2(q_1,\ell)}{a_2^e} f_2^e(a_2^e) + \mathbb{P}(s_2 = h | q_1)\right],$$

$$\psi_{III}(q_1) := \delta\left[\mathbb{P}(s_2 = \ell | q_1)\left(\frac{b_2^e - p_2(q_1,\ell)}{b_2^e - a_2^e} f_2^e(a_2^e) + \frac{p_2(q_1,\ell) - a_2^e}{b_2^e - a_2^e}\right) + \mathbb{P}(s_2 = h | q_1)\right].$$

We then have

$$f_1^e(q_1) = \begin{cases} 0 & \text{if } q_1 < a_1^e, \\ \psi_I(q_1) & \text{if } q_1 \in [a_1^e, \underline{c}_1), \\ \psi_{II}(q_1) & \text{if } q_1 \in [\underline{c}_1, \overline{c}_1), \\ \psi_{III}(q_1) & \text{if } q_1 \in [\overline{c}_1, b_1^e), \\ 1 & \text{if } q_1 \geq b_1^e. \end{cases}$$

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which is equivalent to:

$$\frac{(1-\delta\eta)}{\delta\eta} \geq 1 - \delta\eta + \delta(1-\eta).$$

On the other hand, we saw that in equilibrium the sender is aggressive in period 2 if and only if

$$\frac{1-\delta\eta}{\delta\eta} > \delta\left[2 - \eta - \eta\delta\right].$$

It is now straightforward to check that $1 - \delta\eta + \delta(1-\eta) > \delta\left[2 - \eta - \eta\delta\right]$.

The following properties are easily verified: (i) $\psi_I(0) \geq 0$, (ii) $f_1^e(q_1)$ is a piecewise-affine concave function over $[a_1^e, b_1^e)$, and (iii) $\psi_{III}(b_1^e) < 1$. Concavifying f_1^e then establishes that, in equilibrium, either $M_1(p_1) = \{0, b_1^e\}$ for all $p_1 \in (0, b_1^e)$ (namely, the sender is aggressive in period 1), or else $M_1(p_1) = \{0, a_1^e\}$ for all $p_1 \in (0, a_1^e)$, $M_1(p_1) = \{p_1\}$ for $p_1 \in [a_1^e, d_1^e]$, and $M_1(p_1) = \{d_1^e, b_1^e\}$ for $p_1 \in [d_1^e, b_1^e]$, where $d_1^e \in \{a_1, \underline{c}_1, \overline{c}_1\}$ (namely, the sender is conservative in period 1).

We deduce from the analysis above the following simple characterization of the parametric region such that in equilibrium the sender is conservative in period 1, namely:[20]

$$\frac{a_1^e}{b_1^e} \leq \psi_I(a_1^e). \tag{4}$$

The left-hand side of this inequality represents the sender's expected payoff from splitting $p_1 = a_1^e$ on 0 and b_1^e; the right-hand side represents the sender's expected payoff from supplying no information at $p_1 = a_1^e$. One checks that $\delta \geq \varphi_2(\eta)$ implies (4).

Case B: $\delta < \varphi_2(\eta)$. We divide case B into two subcases. Case B1 is defined by the condition $a_1^e \leq \underline{c}_1$ where, recall, $\underline{c}_1$ is implicitly defined by $p_2(\underline{c}_1, h) = b_2^e$. Case B2 refers to the complementary case, where $a_1^e > \underline{c}_1$. The boundary between the corresponding parametric regions thus satisfies $1 - \underline{c}_1 = \chi_I(\underline{c}_1)$, where

$$\chi_I(q_1) := \delta \left[\mathbb{P}(s_1 = \ell | q_1) \left(\frac{p_2(q_1, \ell)}{b_2^e} b_2^e + 1 - \frac{p_2(q_1, \ell)}{b_2^e} \right) + \mathbb{P}(s_1 = h | q_1) p_2(q_1, h) \right].$$

We analyze case B1 first. Define

$$\chi_{II}(q_1) := \delta \left(\frac{q_1}{b_2^e} b_2^e + 1 - \frac{q_1}{b_2^e} \right).$$

In case B1: $1 - a_1^e = \chi_{II}(a_1^e)$ and $b_1^e = \chi_I(b_1^e)$. These equations give, respectively,

$$a_1^e = \frac{\eta(1-\delta)}{\eta(1+\delta) - 1} \quad \text{and} \quad b_1^e = \frac{\delta \eta^2}{1 - 2\delta\eta(1-\eta)}.$$

[20] Rewriting (4) as $\delta \geq \varphi_I(\gamma)$, in Figure 1 the graph of φ_I corresponds to the lower frontier of the gray region that lies above the dashed curve.

Next, define

$$\psi_{IV}(q_1) := \delta \frac{q_1}{b_2^e},$$
$$\psi_V(q_1) := \delta \left[\mathbb{P}(s_1 = \ell | q_1) \frac{p_2(q_1, \ell)}{b_2^e} + \mathbb{P}(s_1 = h | q_1) \right].$$

Then:

$$f_1^e(q_1) = \begin{cases} 0 & \text{if } q_1 < a_1^e, \\ \psi_{IV}(q_1) & \text{if } q_1 \in [a_1^e, \underline{c}_1), \\ \psi_V(q_1) & \text{if } q_1 \in [\underline{c}_1, b_1^e), \\ 1 & \text{if } q_1 \geq b_1^e. \end{cases}$$

Note that $\psi_{IV}(0) = 0$. Concavifying f_1^e then establishes that, in equilibrium, either $M_1(p_1) = \{0, b_1^e\}$ for all $p_1 \in (0, b_1^e)$ (namely, the sender is aggressive in period 1), or else $M_1(p_1) = \{0, a_1^e\}$ for $p_1 \in (0, a_1^e)$, $M_1(p_1) = \{p_1\}$ for $p_1 \in [a_1^e, \underline{c}_1]$, and $M_1(p_1) = \{\underline{c}_1, b_1^e\}$ for $p_1 \in (\underline{c}_1, b_1^e)$ (namely, the sender is conservative in period 1). In particular, the sender is conservative in period 1 if and only if for $p_1 = \underline{c}_1$ supplying no information ensures an expected payoff at least as large as splitting $p_1 = \underline{c}_1$ on 0 and b_1^e, that is, if and only if[21]

$$\frac{\underline{c}_1}{b_1^e} \leq \psi_V(\underline{c}_1). \tag{5}$$

We next examine case B2. In this case: $1 - a_1^e = \chi_I(a_1^e)$ and $b_1^e = \chi_I(b_1^e)$. These equations give

$$a_1^e = \frac{(1-\delta\eta)\eta}{2\eta + 2\delta(1-\eta)\eta - 1},$$

and b_1^e as in case B1. Moreover:

$$f_1^e(q_1) = \begin{cases} 0 & \text{if } q_1 < a_1^e, \\ \psi_V(q_1) & \text{if } q_1 \in [a_1^e, b_1^e), \\ 1 & \text{if } q_1 \geq b_1^e. \end{cases}$$

Concavifying f_1^e then establishes that, in equilibrium, either $M_1(p_1) = \{0, b_1^e\}$ for all $p_1 \in (0, b_1^e)$

[21]Rewriting (5) as $\delta \geq \varphi_{II}(\gamma)$, in Figure 1 the part of the graph of φ_{II} that lies within the parametric region corresponding to case B1 corresponds to the lower frontier of the gray region that lies below the dashed curve.

(namely, the sender is aggressive in period 1), or else $M_1(p_1) = \{0, a_1^e\}$ for $p_1 \in (0, a_1^e)$, $M_1(a_1^e) = \{a_1^e\}$, and $M_1(p_1) = \{a_1^e, b_1^e\}$ for $p_1 \in (a_1^e, b_1^e)$ (namely, the sender is conservative in period 1), and concludes the proof of Lemma 2. In particular, the sender is conservative if and only if

$$\frac{a_1^e}{b_1^e} \leq \psi_V(a_1^e). \tag{6}$$

One checks that (6) never holds in the parametric region of case B2.

Online Appendix

In this appendix, we provide general conditions under which increasing γ increases the sender's equilibrium expected payoff. In order to do so, we need a few observations and new definitions. Let φ_2 be as defined in the proof of Proposition 2. It follows from (3) that:

1. φ_2 is strictly convex, and continuous, over the interval $[0,1]$;

2. there exists a value $\hat{\gamma}\in(0,1)$ such that φ_2 is decreasing over the interval $[0,\hat{\gamma})$, and increasing over the interval $(\hat{\gamma},1]$,

3. $\max_{\gamma\in[0,1]}\varphi_2=\varphi_2(0)=\varphi_2(1)=1$ and $\min_{\gamma\in[0,1]}\varphi_2=\varphi_2(\hat{\gamma})=\sqrt{3}/2$.

With an abuse of notation, we denote φ_2^{-1} the inverse of the function defined over the interval $[0,\hat{\gamma}]$ and equal to φ_2 over this interval. For each $\delta\in[0,1]$ we define $P(\delta)$ as the, possibly empty, set of values of p_1 for which there exists a $\dot{\gamma}$ and a $\ddot{\gamma}$ such that $\ddot{\gamma}>\dot{\gamma}$ and increasing the value of γ from $\dot{\gamma}$ to $\ddot{\gamma}$ increases the sender's equilibrium expected payoff. Finally, following the notation introduced in the proof of Proposition 2, $\overline{b}_1$ denotes the receiver's period-1 acceptance cutoff assuming the sender is aggressive in period 2.

Proposition 4. *Let $\delta\geq\sqrt{3}/2$. Then:*

1. $(1/2,\overline{b}_1(\varphi_2^{-1}(\delta),\delta))\subseteq P(\delta)$*;*

2. *for δ sufficiently close to $\sqrt{3}/2$,* $(0,\overline{b}_1(\varphi_2^{-1}(\delta),\delta))\subseteq P(\delta)$*.*

Proof: In line with our definition of η, let $\tilde{\eta}(\delta):=\frac{1+\varphi_2^{-1}(\delta)}{2}$ for any $\delta\in[\sqrt{3}/2,1]$. Observations 1-3 above imply that $\tilde{\eta}$ is well defined, and for $\eta<\tilde{\eta}$ in equilibrium the sender is aggressive in period 2, while for $\eta=\tilde{\eta}$ she is conservative.

Let $\underline{c}_1$ be as defined in the Appendix (namely, $\underline{c}_1$ is implicitly defined by $p_2(\underline{c}_1,h)=b_2^e$). The following remark will be used below.

Remark. Let $\delta\geq\sqrt{3}/2$. There exists an $\epsilon>0$ such that $a_1^e\leq\underline{c}_1$ for any η such that $\eta\in[\tilde{\eta}-\epsilon,\tilde{\eta}]$.

We showed in the Appendix, that whenever in equilibrium the sender is conservative in period 2 (we referred to this as Case A) we have $a_1^e=a_1^{\emptyset}<\underline{c}_1$. Continuity of $a_1^{\emptyset}$ and $\underline{c}_1$ in η imply that $a_1^{\emptyset}\leq\underline{c}_1$ also for η sufficiently close to $\tilde{\eta}(\delta)$. As clearly $a_1^e\leq a_1^{\emptyset}$, the remark follows.

Consider the set of parameters for which in equilibrium the sender is aggressive in period 2, and $a_1^e \leq \underline{c}_1$. For these parameters, in equilibrium the sender is aggressive in period 1 if and only if:

$$\frac{a_1^e}{b_1^e} \geq \delta \frac{a_1^e}{b_2^e} \Leftrightarrow \frac{a_1^e}{\overline{b}_1} \geq \delta \frac{a_1^e}{b_2^\emptyset} \Leftrightarrow \delta \leq \frac{b_2^\emptyset}{\overline{b}_1}.$$

As $\overline{b}_1 = \frac{\delta\eta^2}{1-2\delta\eta(1-\eta)}$, the last inequality is equivalent to $\delta \leq \frac{1}{\eta(3-2\eta)}$. Note that

$$min_{\eta\in[1/2,1]} \frac{1}{\eta(3-2\eta)} = 8/9 > \sqrt{3}/2.$$

Fix $\delta \in (\sqrt{3}/2, 8/9)$, so that for any $\eta < \tilde{\eta}(\delta)$ such that $\tilde{\eta}(\delta) - \eta$ is sufficiently small, in equilibrium the sender is aggressive in period 1, and $b_1^e = \overline{b}_1$. Also, let $p_1 \in (0, \overline{b}_1(\tilde{\eta}(\delta), \delta))$. As $\overline{b}_1$ is continuous and increasing in η, and $\overline{b}_1(\tilde{\eta}(\delta), \delta) > b_1^e(\tilde{\eta}(\delta), \delta)$, there exists some η' such that $\eta' < \tilde{\eta}(\delta)$ and $b_1^e(\eta', \delta) = \overline{b}_1(\eta', \delta) > \max\{p_1, b_1^e(\tilde{\eta}(\delta), \delta)\}$. Hence an increase in η from η' to $\tilde{\eta}(\delta)$ increases the sender equilibrium expected payoff from $p_1/\overline{b}_1(\eta', \delta)$ to an expected payoff at least as large as $\min\{1, p_1/b_1^e(\tilde{\eta}(\delta), \delta)\}$. Part 2 of the proposition follows.

In order to prove part 1, note first that for any $\delta \in [\sqrt{3}/2, 1]$ either there exists a sequence of $\eta < \tilde{\eta}(\delta)$ converging to $\tilde{\eta}(\delta)$ such that in equilibrium the sender is aggressive in period 1 for every η, or else there exists a sequence of $\eta < \tilde{\eta}(\delta)$ converging to $\tilde{\eta}(\delta)$ such that in equilibrium the sender is conservative in period 1 for every η. In the former case, the proof of case 2 implies $(1/2, \overline{b}_1(\varphi_2^{-1}(\delta), \delta)) \subseteq P(\delta)$. We focus now on values of δ for which the latter case holds.

Note that $\underline{c}_1 < 1/2$ for any δ and any η, and $\underline{c}_1$ is a continuous function of δ and η. The remark above and the equilibrium characterization in the Appendix (see case B1) ensure that for any η such that $\eta < \tilde{\eta}(\delta)$, $\tilde{\eta}(\delta) - \eta$ is sufficiently small and in equilibrium the sender is conservative in period 1, the sender's expected payoff for any $p_1 \in (1/2, \overline{b}_1(\eta, \delta))$ equals (again, see the Appendix)

$$\frac{\overline{b}_1(\eta, \delta) - p_1}{\overline{b}_1(\eta, \delta) - \underline{c}_1} \psi_V(\underline{c}_1) + \frac{p_1 - \underline{c}_1}{\overline{b}_1(\eta, \delta) - \underline{c}_1}. \tag{7}$$

As ψ_V is a continuous function of η, so is (7).

Let $p_1 \in (1/2, \overline{b}_1(\tilde{\eta}(\delta), \delta))$. There exists a sequence of η_n such that $\lim_{n\to\infty} \eta_n = \tilde{\eta}(\delta)$ and, for every n:

- $\eta_n < \tilde{\eta}(\delta)$;

- the sender is conservative in period 1 for $\eta = \eta_n$;
- $b_1^e(\eta_n, \delta) = \overline{b}_1(\eta_n, \delta) > \max\{p_1, b_1^e(\tilde{\eta}(\delta), \delta)\}$.

It ensues that

$$\lim_{n\to\infty} \frac{\overline{b}_1(\eta_n,\delta) - p_1}{\overline{b}_1(\eta_n,\delta) - \underline{c}_1(\eta_n,\delta)} \psi_V(\underline{c}_1(\eta_n,\delta)) + \frac{p_1 - \underline{c}_1(\eta_n,\delta)}{\overline{b}_1(\eta_n,\delta) - \underline{c}_1(\eta_n,\delta)} =$$
$$\frac{\overline{b}_1(\tilde{\eta}(\delta),\delta) - p_1}{\overline{b}_1(\tilde{\eta}(\delta),\delta) - \underline{c}_1(\tilde{\eta}(\delta),\delta)} \psi_V(\underline{c}_1(\tilde{\eta}(\delta),\delta)) + \frac{p_1 - \underline{c}_1(\tilde{\eta}(\delta),\delta)}{\overline{b}_1(\tilde{\eta}(\delta),\delta) - \underline{c}_1(\tilde{\eta}(\delta),\delta)} <$$
$$\frac{b_1^e(\tilde{\eta}(\delta),\delta) - p_1}{b_1^e(\tilde{\eta}(\delta),\delta) - \underline{c}_1(\tilde{\eta}(\delta),\delta)} \psi_V(\underline{c}_1(\eta,\delta)) + \frac{p_1 - \underline{c}_1(\tilde{\eta}(\delta),\delta)}{b_1^e(\tilde{\eta}(\delta),\delta) - \underline{c}_1(\tilde{\eta}(\delta),\delta)}, \tag{8}$$

where the inequality holds as $b_1^e(\tilde{\eta}(\delta), \delta) < \overline{b}_1(\tilde{\eta}(\delta), \delta)$ and $\psi_V(\underline{c}_1) < 1$ for any $\underline{c}_1 \in [0, 1/2]$.

Expression (8) is the expected payoff that the sender can ensure for herself by being conservative in period 1 for $\eta = \tilde{\eta}(\delta)$. We then conclude that for sufficiently large n and increase in η from η_n to $\tilde{\eta}$ increases the sender's equilibrium expected payoff. Part 2 of the proposition follows.

■

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