Employment, Output and Welfare Effects of Minimum Wages[*]

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Abstract

Many countries are discussing substantial increases in the minimum wage. However, policy makers lack a comprehensive analysis of the macroeconomic implications of raising the minimum wage. This paper investigates how employment, output and welfare respond to increases in the minimum wage beyond observable levels – both in the short- and long run. To that end, I incorporate endogenous job search effort, differences in employment levels, and a progressive tax-transfer system into a search-matching model with worker and firm heterogeneity. I estimate my model using German administrative and survey data. The model can capture the muted employment response, as well as the reallocation effects in terms of productivity and employment levels found by reduced form research on the German introduction of a federal minimum wage in 2015. Simulating the model, I find that long-run employment increases slightly until the minimum wage is equal to 60% of the full-time median wage (Kaitz index) as higher search effort offsets lower vacancy posting. In addition, raising the minimum wage reallocates workers towards full-time jobs and high-productivity firms. Total hours worked and output peak at Kaitz indices of 73% and 79%. However, policy makers face an important inter-temporal trade-off as large minimum wage hikes lead to substantial job destruction, unemployment and recessions in the short-run. Finally, not all workers benefit equally from higher minimum wages. For women, who often rely on low-hours jobs, the disutility from working longer hours outweighs the utility of higher incomes. Moreover, high minimum wages force low-skill workers into long-term unemployment.

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1   Introduction

The minimum wage is one of the most frequently used labor market policies in developed countries. In the benchmark model of fully competitive labor markets, wages equal marginal productivity and a binding minimum wage always reduces employment, output and welfare. However, a large body of empirical research has found only very muted employment effects for observed minimum wages ranging between 30 and 60% of the full-time median wage (Kaitz index) (Dube, 2019).[1] In addition, recent evidence shows that minimum wages not only increase earnings but also improve the quality of jobs by reallocating workers towards high-productivity firms and jobs with higher employment levels (Dustmann et al., 2020). Against this backdrop, many countries are discussing proposals to substantially increase the minimum wage, but policy makers lack a comprehensive analysis of the macroeconomic and distributional implications of raising the minimum wage beyond observed levels.[2]

This paper takes a first step towards filling this gap. Specifically, I use a rich search-and-matching model in order to analyze how the minimum wage affects employment, output and welfare. I first estimate the model using German administrative and survey data from 2014 – the year before Germany introduced a federal minimum wage that affected more than ten percent of jobs. Second, I evaluate the macroeconomic and distributional implications of the German minimum wage reform and show that the model is consistent with recent reduced-form evidence on the reform's short-run employment and productivity effects (e.g. Dustmann et al., 2020). Finally, I use the estimated model to quantify the short-run and long-run effects of a hypothetical reform that raises the minimum wages well beyond the current level in Germany.

The analysis is based on a search-and-matching model of the labor market with substantial worker and firm heterogeneity, differences in employment levels, and a progressive tax-and-transfer system. In the model, the effect of the minimum wage on employment is ambiguous since firms' vacancy posting and workers' job search decisions are affected in opposite directions (Flinn, 2006; Acemoglu, 2001). On the one hand, firms will lower their vacancy creation as the minimum wage cuts into match profits. On the other hand, the minimum wage increases wages, earnings and thus the surplus of finding a job, which leads workers to exert more search effort. The net effect on employment is therefore a quantitative question.[3]

In addition to the employment effect, minimum wages also affect output by changing the *composition* of jobs along two dimensions. First, raising the minimum wage increases average productivity because profits and thus vacancy posting decline more strongly for low-productivity

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[1] The majority of high-income OECD – including for example the US, Canada, Japan, South Korea or Australia and 21 out of 27 European countries countries – has a minimum wage in place. The Kaitz index varies between 30 and 60 percent. This variation is also present within the US where state-level minimum wages vary between the federal minimum of 7.25 USD and 14 USD.

[2] For example, US Democrats have proposed federal minimum wage of 15 USD (Kaitz index $\approx 75\%$). In Germany, there is a discussion about raising the minimum wage to 12 EUR ($\approx 62\%$). The Polish government plans a 73% increase over and the UK government officials plan to raise the minimum wage to 67% of the median over. The Italian government plans to introduce a minimum wage.

[3] Acemoglu (2001) briefly discusses this possibility, but does not analyze which channel dominates quantitatively. Bagger and Lentz (2018) allow for endogenous search effort in their analysis of firm-worker sorting. Outside the minimum wage literature, the idea that the surplus of employment affects workers' search effort and employment is standard in the literature on unemployment benefits (e.g. Meyer, 1990; Chetty, 2008; Schmieder et al., 2012; Marinescu and Skandalis, forthcoming).

firms (Eckstein and Wolpin, 1990; Acemoglu, 2001). Second, raising the minimum wage increases the average employment level, as was recently documented for example by Dustmann et al. (2020). In particular, my model allows for three different employment levels, which I call full-time, part-time and marginal jobs. Differences in employment levels are particularly important as most tax- and transfer systems in developed countries subsidize low-earnings jobs. As a result, low-hours jobs are concentrated in the bottom part of the wage distribution and become relatively less profitable in the presence of a binding minimum wage.[4] The shift towards full-time jobs is amplified by the fact that workers' incentives to search for full-time jobs increase in the hourly wage.

The analysis proceeds in three steps. In a first step, I estimate the model using German administrative linked employer-employee as well as survey data from 2014, i.e. the last year where the economy was not distorted by a federal minimum wage. I show that the estimated model is able to match the joint distribution of wages, firm productivity and employment levels. This is important as it determines the scope for reallocation and thus output effects of increasing the minimum wage. The model also matches the distribution of labor market states across demographics which allows for an analysis of heterogeneous welfare effects.

In the second step, I assess the macroeconomic implications of the introduction of a federal minimum wage in Germany in 2015. I find that the introduction of a minimum wage of 8.5 EUR (Kaitz index of 47%) had negligible employment effects, but led to an increase in average hours worked and firm productivity of 1.4% and 0.6% respectively. The model predicts that this change in the composition of jobs increased output by 0.4% over the first five years and will increase output by almost 0.5% in the long-run. However, I also find that the German tax- and transfer-system prevents consumption growth from keeping up with earnings growth. Higher earnings reduce the level of transfer payments workers receive. Together with higher disutility of longer working hours, this implies that the welfare gains of the reform are negligible. Nevertheless, workers are now less reliant on government transfers to top up their earnings. Importantly, the model's short-run predictions of a null-effect on total employment, a shift from marginal to part-time and full-time jobs, and an increase in average firm productivity are qualitatively and quantitatively consistent with the short-run effects documented by recent reduced-form studies (e.g. Garloff, 2016; vom Berge et al., 2016; Caliendo et al., 2017; Dustmann et al., 2020). The fact that the model is consistent with the reduced-form evidence on a large and observed minimum wage reform lends credibility to the following analysis of counterfactual minimum wage levels.

In the third and most important step, I analyze how raising the minimum wage well beyond observable levels affects employment, output and welfare. Importantly, I analyze not only the new stationary equilibrium but the entire transition path. Focusing on the long-run effects, I find that total employment, i.e. the number of jobs, slightly increases in the minimum wage up until a Kaitz index of 60% (11 EUR) as higher search effort outweighs lower vacancy posting.[5] As the minimum wage increases further, the reduction in vacancies dominates and total employment

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[4]In Germany in 2014, full-time jobs accounted for only one third of the jobs affected by the initial minimum wage which affected more than ten percent of all jobs.

[5]While there is no evidence for Germany, Cengiz et al. (2019) find no disemployment effects for past US minimum wage reforms with Kaitz of up to 60%.

starts to fall. I further find that raising the minimum wage can substantially increase total output as the share of low-hours and low-productivity jobs monotonically decreases in the minimum wage. Total hours worked are maximized at a Kaitz index of 73% (13.5 EUR). At the long-run output maximum at a Kaitz index of 79% (14.5 EUR), total output is about 3.6% above the baseline level. Average firm productivity and total hours worked are 3.3% and 4.0% above the baseline level respectively. Quantitatively, the increase in average output per job more than offsets the lower number of low-skill jobs whose contribution to total output is relatively small.[6]

In addition to the steady-state analysis, this paper goes beyond the existing literature by analyzing the entire transition path. The results show that short- and long-run effects differ significantly. Specifically, a sudden increase in the minimum wage will cause an initial drop in employment even if employment hardly changes in the long-run. The larger the increase in the minimum wage, the more workers initially lose their job because it has become unprofitable for the firm. Search frictions imply that it takes time for the unemployment rate to drop again. This is amplified by the fact that firms now post substantially fewer vacancies. For minimum wages above 60% of the median wage, output declines on impact. At the long-run output optimum, the unemployment rate initially more than doubles and, on average, is about 60% (45%) higher over the first two (five) years after the minimum wage hike. As a result, the economy goes through a recession of almost two years.

Finally, I show that the minimum wage does not benefit all workers equally. Women, who tend to prefer jobs with fewer weekly hours, experience increasing disutility from work as firms offer fewer vacancies for low-hours jobs. This disutility outweighs the utility gains from higher consumption.[7] In addition, low-skill workers become non-employable and are stuck in long-term unemployment as firms will no longer hire them at the minimum wage.

In sum, this paper makes three contributions. First, I incorporate endogenous job search effort, differences in employment levels, and a progressive tax-transfer system into a search-matching model with worker and firm heterogeneity and show that the estimated model matches the joint distribution of wages, firm productivity and employment levels. Second, I use the estimated model to assess the macroeconomic and distributional implications of the introduction of a federal minimum wage in Germany in 2015 and show that the model is consistent with the available, empirical evidence. Third and most importantly, I provide a comprehensive analysis of the short- and long-run impact on employment, output and welfare of raising the minimum wage beyond observable levels.

Related Literature. My paper speaks to several strands of the literature. Most importantly, my paper adds to the large literature investigating the effects of minimum wages in labor markets with search frictions. Some early contributions assume that contact rates are exogenously given and not affected by the minimum wage in wage posting models (Burdett and Mortensen, 1998;

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[6]I assume that there are no skill-complementarities in production. This assumption is supported by the findings of Cengiz et al. (2019) who demonstrate that the minimum wage elasticity for higher-skilled employment should be very small with a neoclassical production function and plausible parameter values for the elasticity of substitution between low- and high-skill workers. This is mainly driven by the small output share of minimum wage workers. In addition, they find no evidence for labor-labor substitution.

[7]Note that I interpret this disutility as a rather general proxy capturing not only the utility of leisure but also outside constraints such as childcare obligations.

Bontemps et al., 1999; van den Berg and Ridder, 1998). Both Eckstein and Wolpin (1990) and Acemoglu (2001) allow for endogenous vacancy creation and show theoretically that a minimum wage induces a trade-off between the total number of jobs and their average productivity. Flinn (2006) estimates a stylized search-matching model with endogenous contact rates in which the employment effect of minimum wages need not be negative even though firms can adjust vacancy posting. Engbom and Moser (2018) estimate a wage-posting model with worker and firm heterogeneity as well as endogenous vacancy creation in order to quantify the contribution of an increase in the minimum wage to the decline of wage inequality in Brazil. In simultaneous and independent work, Blömer et al. (2020) estimate the wage posting model by Bontemps et al. (1999) to analyze minimum wage effects on steady state full-time employment in Germany.[8] I contribute to this literature by quantifying employment effects when both vacancy posting and search effort are optimally chosen by firms and workers.[9] In addition, my paper is the first to analyze how minimum wages affect output when jobs differ not only by firm productivity but also employment level.[10] My model also differs by allowing for a progressive tax- and transfer system that subsidizes low-earnings jobs, as is the case in most developed countries. This is important for our understanding of reallocation effects as it shapes the joint distribution of employment levels and wages. Finally, this is the first paper to analyze transition dynamics of minimum wage hikes and show that policy makers have to weigh long-run output and welfare gains of higher minimum wages against severe short-term unemployment.

I further contribute to the literature evaluating past minimum wage reforms which mostly consists of reduced-form papers. Harasztosi and Lindner (2019) analyze who pays for the minimum wage in Hungary. Portugal and Cardoso (2006) and Dube et al. (2016) show that minimum wages reduce employment flows. Cengiz et al. (2019) provide an extensive analysis of employment effects of past minimum wage reforms in the US. The short-run effects of the German minimum wage reform of 2015 is analyzed most prominently by Dustmann et al. (2020) as well as e.g. Garloff (2016), Caliendo et al. (2017), Holtemöller and Pohle (2017), and Burauel et al. (2020). This paper's structural approach is able to add a macroeconomic perspective by analyzing output and welfare effects. In addition, the model with endogenous search effort can rationalize why reduced-form studies have not found significant disemployment effects even for high levels of the minimum wage (e.g. Cengiz et al., 2019; Dustmann et al., 2020).

Finally, by including endogenous search effort, my paper is also related to the literature on employment effects of other labor market policies that target the surplus of employment. The large literature on unemployment benefits has worked to understand how benefits or benefit

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[8]Apart from my aforementioned general contributions, my paper differs along a number of dimensions. First, Blömer et al. (2020) only focus on full-time employment which accounts for only one third of the jobs affected by the initial minimum wage of 8.5 EUR and less than half of the jobs between 8.5 and 12.5 EUR in 2014. I show that taking non-full-time work into account is important for understanding the reallocation effects of the minimum wage. Second, they do not discuss transition dynamics which I show to be of great importance when assessing a minimum wage hike. Thirdly and most importantly, they focus solely on the unemployment rate while my paper provides a joint analysis of employment, output and welfare effects of increasing the minimum wage.

[9]While Acemoglu (2001) theoretically shows that endogenous search effort can mute disemployment effects of minimum wages, he does not quantify the contribution of this channel in an estimated model. Bagger and Lentz (2018) use endogenous search effort to explain sorting of workers and firms. Krebs and Scheffel (2016) use a search matching model with endogenous search to evaluate the German Hartz reforms.

[10]A recent paper by Doppelt (2019) shows theoretically and using reduced form evidence that higher minimum wages lead workers to work longer hours. However, he does not embed the mechanism in a quantitative model to analyze output effects.

duration affect employment by influencing workers' incentives to exert search effort and find a job (e.g. Ljungqvist and Sargent, 1998; Chetty, 2008; Krebs and Scheffel, 2013; Schmieder et al., 2016). There is also a literature in macroeconomics analyzing unemployment insurance policies over the business cycle in search-matching models (e.g. Mortensen and Pissarides, 1994; Krause and Uhlig, 2012; Hagedorn et al., 2019; Mitman and Rabinovich, 2019). While these papers study how the surplus of employment evolves when unemployment benefits change, I analyze how minimum wages affect employment because the value of employment is affected by the minimum wage.[11] My paper thus suggests that unemployment benefits and minimum wages interact and should potentially be set jointly.

Outline. The remainder of the paper is structured as follows. Section 2 presents the equilibrium search-matching model. Section 3 describes the parameterization, identification and estimation of the model and evaluates the model fit. Section 4 analyzes the introduction of the German minimum wage. Section 5 analyzes counterfactually high minimum wages. Finally, section 6 discusses the results and concludes.

2 Model

2.1 Workers

Workers are allowed to differ by gender and family status. In particular, I distinguish between the following five sociodemographic groups indexed by j: married men, single men, single women with and without kids, and married women (see Table 1).[12] Let p_j denote the population share of group j. A worker's sociodemographic type determines her preferences over employment levels as well as her tax-and transfer schedule.[13]

Workers further differ by their time-invariant human capital (skill) h. The gender-specific distribution function of human capital is $\Phi^{g(j)}$ where g is the gender of group j. I assume that

TABLE 1: Sociodemographic Types

	$\Pr(j)$	$\Pr\left(g(j)\right)$	$\Pr\left(j\vert g(j)\right)$
Sociodemographics			
Men, Single	0.214	0.514	0.416
Men, Married	0.300	0.514	0.584
Women, Single, No Kids	0.168	0.486	0.346
Women, Single, Kids	0.046	0.486	0.095
Women, Married	0.272	0.486	0.560

Note: The share of each sociodemographic group conditional on gender $g(j)$ is computed from the SOEP and then multiplied by the respective gender share in the SIAB data. Source: SOEP, SIAB, own calculations.

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[11]In a recent paper by Hartung et al. (2018), the value of unemployment not only affects job finding rates but also separation rates as it leads workers to accept lower wages in return for greater job stability.

[12]As men with and without children are similar with respect to all targeted moments, I only distinguish between single and married men. The same holds for married women.

[13]Whenever possible, I will drop the subscript j for worker types to improve readability.

the labor market is segmented with respect to workers' skill levels such that there is a continuum of independent labor markets – one for each level of h (van den Berg and Ridder, 1998; Engbom and Moser, 2018).

A type-j worker with human capital h can be employed, $s = e$, short-term unemployed, $s = su$ or long-term unemployed, $s = lu$. There are three employment *levels* which I label full-time, part-time and marginal employment, $x \in \{f, p, m\}$. In addition, jobs differ with respect to the employer's productivity p which will be described below. While short-term unemployed workers receive unemployment insurance proportional to their previous earnings, all long-term unemployed workers receive the same unemployment benefits, i.e. a subsistence minimum. In sum, for each skill level h there is a continuum of idiosyncratic states for employed and short-term unemployed workers and a single state for long-term unemployment. The state space of a type-j worker with human capital h is

$$\mathcal{S} = \Big\{ \{(s, x, p) \mid s \in \{e, su\}, x \in \{f, p, m\}, p \geq 1\}, lu \Big\}$$

In the following I denote by σ one point in the state-space of a worker and F the distribution of endogenous states (given j and h).

When a worker with human capital h works a type-x job at a firm with productivity p, the match output is $f(h, x, p) = e_x a_x h p$. The parameters $a_x > 0$ allow for constant productivity differences between full-time, part-time and marginal jobs. Workers earn a fixed and exogenous share $r \in (0, 1)$ of the match output.[14] In the presence of a minimum wage $\bar{w}$, the hourly wage is

$$w(h, x, p) = \max\{rf(h, x, p), \bar{w}\} \tag{1}$$

Gross earnings and net earnings are given by

$$\tilde{y}(h, x, p) = e_x w(h, x, p)$$
$$y^j(h, x, p) = \tilde{y}(h, x, p) - T^j(\tilde{y}(h, x, p)) \tag{2}$$

where $T^j(\tilde{y})$ is a tax function that depends on the worker's sociodemographics.[15]

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[14]There are a number of reasons for not using a more involved wage setting mechanism such as Nash bargaining (Cahuc et al., 2006) or wage posting Burdett and Mortensen (1998). First, not having to solve for a wage-posting schedule or bargained wage keeps the estimation of the model feasible as the combination of endogenous worker search effort, and multiple worker types and employment levels makes the computation of the equilibrium time-consuming. Second, match-level wage determination in search-matching models remains a black box and little is known about the validity of the wage-posting or bargaining assumptions. While certainly too simple, the assumption of an exogenous piece rate ensures that (i) I match the aggregate labor share and (ii) the results are not driven by a poorly-understood mechanism. Third, recent evidence by Jäger et al. (2020) shows that – even for previously unemployed workers – wages are insulated from the value of non-employment. Fourth, a recent paper by Di Addario et al. (2020) finds that a core prediction of the sequential auction model (Postel-Vinay and Robin, 2002; Bagger et al., 2014) is not supported by Italian social security data. In particular, the productivity of the firm where the worker is poached/hired from has almost no effect on the wage at the destination firm. Fifth, wage posting implies substantial wage spillover which have not been found by Cengiz et al. (2019) and Dustmann et al. (2020). Finally, a fixed piece-rate could be motivated by Nash bargaining over the match output instead of the match surplus.

[15]I refer to taxes as the sum of income taxes and social security contributions.

Short-term unemployed workers receive a share b of their previous net earnings up to a maximum amount of B_{max} (unemployment insurance). Long-term unemployed workers receive subsistence benefits B_{min} independent of their skill level or previous earnings. Short-term unemployment insurance is capped from below by B_{min}. Employed workers are also eligible for unemployment benefits to top up their net earnings or unemployment insurance. In doing so, a share τ_{top} of net earnings will be deducted from B_{min}. Finally, married workers receive non-labor income y_{free}^j which is always deducted from B_{min}.[16] Hence, subsistence benefits for type-j workers may not exceed $B_{min}^j = \max\{B_{min} - y_{free}^j, 0\}$.

As there is no savings device, consumption c equals net income. A type-j worker with skill h faces the following consumption schedule:

$$c^j(h, \sigma) = \begin{cases} y^j(h, x, p) + \max\left\{B_{min}^j - \tau_{top} y^j(h, x, p), 0\right\} + y_{free}^j & \text{if } s = e \\ by^j(h, x, p) + \max\{B_{min}^j - by^j(h, x, p), 0\} + y_{free}^j & \text{if } s = su \\ B_{min}^j + y_{free}^j & \text{if } s = lu \end{cases} \quad (3)$$

Workers exert costly search effort ℓ to find (better) jobs in their skill-segment of the labor market. A worker in employment state s meets a vacancy with probability

$$\lambda_\sigma(\ell|h) = \phi^\sigma \ell \Lambda(\theta_h) \quad (4)$$

where labor market tightness θ_h is taken as given and ϕ^σ is a search efficiency parameter. I will assume that search efficiency differs by employment level and between short- and long-term unemployed ($\phi^{su}, \phi^{lu}, \phi^{e_x}$). Importantly, not every meeting has to result in a match as search cannot be directed towards certain employment levels or high-productivity firms, and workers may decline lower-valued offers.

The mass of search-weighted workers of type-j is denoted by $S^j(h)$ and the mass of all search-weighted workers in skill segment h is

$$S(h) = \sum_j p_j \underbrace{\int_\sigma \phi^\sigma \ell(\sigma|j, h) \mathrm{d}F(\sigma|j, h)}_{S^j(h)} \quad (5)$$

where $\ell(\cdot|j, h)$ and $F(\cdot|j, h)$ represent the optimal search effort and stationary distribution functions for type-j workers in skill segment h.

Workers' utility depends on consumption, the employment level and job search:

$$u^j(\ell|h, \sigma) = \tilde{u}(c^j(h, \sigma)) - d(\ell) + \nu^j(x(\sigma)) \quad (6)$$

Thereby, $\tilde{u}(c)$ is a concave flow utility function of consumption, $d(\ell)$ is a convex search cost function and $\nu^j(x(\sigma))$ captures the (dis-)utility of different employment levels relative to nonemployment. The latter may depend on workers' sociodemographics j. Single women with kids may for

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[16]The type-specific and exogenous non-labor income y_{free}^j represents a share of the partner's income for married workers. Singles do not receive such non-labor income.

example have a strong preference for part-time or marginal jobs.[17] Heterogeneity in $\nu^j(x)$ will allow the model to match the joint distribution of employment levels and sociodemographics.

2.2 Firms

There is a mass m_f of risk-neutral firms with heterogeneous productivity $p \sim \Gamma$. Firms employ workers of all skill levels h at all employment levels x. As is standard in the literature (e.g. Bagger and Lentz, 2018; Engbom and Moser, 2018), I assume that firms operate a linear production technology such that total output of a firm with productivity p is the sum of the match outputs

$$\sum_x \int_{\underline{h}}^{\bar{h}} f(h, x, p) L(h, x, p) \mathrm{d}h$$

where $L(h, x, p)$ is the firm's mass of employees with skill h and demographics j working a type-x job.[18]

Firms attract workers for type-x jobs in skill segment h by posting vacancies $v(h, x)$ at a convex cost $\kappa_x(h, v)$. As hiring a worker does not affect future recruitment, firms will not reject workers of a particular demographic type even if different workers are more or likely to switch employers than others. Denote by $N(h, x)$ the mass of type-x vacancies in skill segment h and the total number of vacancies as $N(h) = \sum_x N(h, x)$. In addition, let $\Psi(h)$ denote the distribution of employment levels and productivities among all vacancies in skill segment h. Firms' vacancy posting response to a binding minimum wage can affect both the $N(h)$ and $\Psi(h)$. The former impacts labor market tightness, job finding probabilities and the total number of jobs. The latter will determine the composition of jobs and thus the average productivity and employment level.

2.3 Labor Market

Recall that labor markets are segmented by worker skill h and workers cannot direct search towards a certain employment level or towards high-productivity firms. Hence, the total mass of search and vacancies in a skill-segment are matched by the matching function

$$M(h) = N(h)^\xi S(h)^{1-\xi} \tag{7}$$

where ξ is the elasticity of matches with respect to the mass of posted vacancies. Labor market tightness is defined as

$$\theta(h) = \frac{N(h)}{S(h)} \tag{8}$$

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[17]I emphasize that these "preference" parameters not only capture the tastes for leisure, but also exogenous constraints such as childcare obligations. As I do not explicitly model policies affecting child care constraints, using such a proxy is justified even though the parameter is not policy-invariant outside the model.

[18]I assume that there are no skill-complementarities in production. This assumption is supported by the findings of Cengiz et al. (2019) who demonstrate that the minimum wage elasticity for higher-skilled employment should be very small with a neoclassical production function and plausible parameter values for the elasticity of substitution between low- and high-skill workers. This is mainly driven by the small output share of minimum wage workers. In addition, they find no evidence for labor-labor substitution.

and the aggregate contact rates for a unit of search and a vacancy are $\Lambda(\theta) = \theta^\xi$ and $\Pi(\theta) = \theta^{\xi-1}$ respectively.

Employment relationships are terminated for three mutually exclusive reasons. First, workers may voluntarily change firms and/or employment levels as a result of on-the-job search. In equilibrium, firms with low productivity will be more likely to experience this event.

Second, workers may be hit by a so-called Godfather shock which forces them to switch to a different job that is randomly drawn from the distribution of vacancies. This is important to account for the substantial share of job-to-job transitions that are accompanied by a wage cut and cannot be explained by on-the-job search (Jolivet et al., 2006). The Godfather shock arrives with probability $\pi_{e|e_x}(h) = \psi_x \Lambda(\theta)$ and captures involuntary and unintended job-to-job transitions unrelated to workers' search effort. These may be the result of firms' outplacement programs, workers' search effort after an advance-notice layoff or family-related events that force workers to move and look for a new job immediately.

Third, matches can be destroyed such that the worker transitions into short-term unemployment. This happens with probability $\pi_{su|e_x}$ and if a minimum wage hike makes the match unprofitable for the firm.

2.4 Worker Problem

Workers choose search effort ℓ and reject or accept job offers in order to maximize discounted lifetime utility. Labor market tightness and the distribution of vacancies are taken as given.

The value of long-term unemployment for a type-j worker with human capital h solves the following Bellman equation:

$$V_{lu}^j(h) = \max_\ell \left\{ u^j(\ell, h, lu) + \beta \lambda_{lu}(\ell|h) \mathbb{E}_{(x,p)}\left[\max\left\{ V_e^j(h, x, p), V_{lu}^j(h) \right\} \big| h \right] \right. \\ \left. + \beta\big(1 - \lambda_{lu}(\ell|h)\big) V_{lu}^j(h) \right\} \tag{9}$$

Search effort ℓ is associated with lower flow utility but a higher probability of meeting a firm. Upon on meeting a firm offering a (x, p) job, the worker accepts the job if and only if the value of the employment relationship, $V_e^j(h, x, p)$, exceeds the value of remaining long-term unemployed. The max-operator in the continuation value captures this acceptance decision. The expectation is taken with respect to the distribution of vacancies in the worker's skill segment. With probability $1 - \lambda_{lu}(\ell|h)$, the worker does not meet a firm and remains long-term unemployed.

The value of short-term unemployment when the previous job was of type x at a type-p firm is

$$V_{su}^j(h, x, p) = \max_\ell \left\{ u^j\big(\ell|h, (su, x, p)\big) + \beta \pi_{lu|su} V_{lu}^j(h) \right. \\ + \beta \lambda_{su}(\ell|h) \mathbb{E}_{(x',p')}\left[\max\left\{ V_e^j(h, x', p'), V_{su}^j(h, x, p) \right\} \big| h \right] \\ \left. + \beta\big(1 - \pi_{lu|su} - \lambda_{su}(\ell|h)\big) V_{su}^j(h, x, p) \right\} \tag{10}$$

The only difference to long-term unemployment is that the worker transitions from short- to long-term unemployment with exogenous probability $\pi_{lu|su}$.

The value of a worker employed at a type-p firm on a type-x job is

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\begin{aligned}
V_e^j(h, x, p) = \max_\ell \Big\{ & u^j\big(\ell|h, (e, x, p)\big) + \beta \pi_{su|e_x} V_{su}^j(h, x, p) \\
& + \beta \lambda_{e_x}(\ell|h) \mathbb{E}_{(x', p')}\Big[\max\big\{ V_e^j(h, x', p'), V_e^j(h, x, p)\big\} \big| h \Big] \\
& + \beta \pi_{e|e_x}(h) \mathbb{E}_{(x', p')}\Big[V_e^j(h, x', p') \big| h \Big] \\
& + \beta\big(1 - \pi_{su|e_x} - \lambda_e(\ell|h) - \pi_{e|e_x}(h)\big) V_e^j(h, x, p) \Big\}
\end{aligned}
\tag{11}
$$

Employed workers become short-term unemployed with probability $\pi_{su|e_x}$, receive a job offer that they can decline through on-the-job search with probability $\lambda_{e_x}(\ell|h)$ and are involuntarily reallocated to a different job with probability $\pi_{e|e_x}(h)$.

All workers may have an incentive to search for a (better) job. The first order condition determining optimal search effort is given by

$$
\frac{\mathrm{d}d^j(\ell)}{\mathrm{d}\ell} = \beta \frac{\partial \lambda_\sigma(\ell|h)}{\partial \ell} \Big(\underbrace{\mathbb{E}_{(x,p)}\Big[\max\big\{ V_e^j(h, x, p), V^j(h, \sigma)\big\} \big| h \Big] - V^j(h, \sigma)}_{\text{expected surplus of meeting a firm}} \Big)
\tag{12}
$$

For a worker in state σ, the job finding probability is the result of optimal search effort $\ell(\sigma)$ as well as the worker's acceptance decision

$$
\pi^j(\ell|h, \sigma) = \lambda_\sigma\big(\ell|h\big) \mathbb{E}_{(x,p)}\Big[\mathbb{1}\{ V_e^j(h, x, p) > V^j(h, \sigma)\} \big| h \Big]
\tag{13}
$$

2.5 Firm Problem

Firms maximize expected discounted profits taking as given labor market tightness, the distribution of vacancies and the distribution of workers' search effort. As total production is additive in h and x, the firm faces a sequence of independent optimization problems – one for each (h, x)-segment. Each period, firms post vacancies which may result in an employment relationship starting in the subsequent period. Unfilled vacancies are not carried over to the next period but have to be re-posted. Additive production combined with the fact that the cost of posting vacancies is independent of the current workforce further implies that the firm's optimal amount of vacancies is independent of the current workforce. For the same reasons, firms will not reject workers of a particular demographic type.

A type-x employment relationship with a type-j employee may be dissolved either due to exogenous job destruction, a Godfather shock or on-the-job search with probability:

$$
\delta^j(h, x, p) = \pi_{su|e_x} + \pi_{e|e_x}(h) + \pi^j(\ell(\sigma)|h)
\tag{14}
$$

The probability of filling a vacancy is equal to the aggregate contact rate times the probability that the contacted worker accepts the offer:

$$\eta(h, x, p) = \Pi(\theta_h) \frac{S(h, x, p)}{S(h)} \tag{15}$$

Thereby, $S(h)$ is the total search-weighted mass of workers in skill segment h and $S(h, x, p) = \sum_j S^j(h, x, p)$ is the mass of search-weighted workers in segment h willing to accept a type-x job at a firm with productivity p.

Let $(1 - r^+)$ be the firm's profit share of the match output. If the minimum wage is binding for a (h, x, p)-job, $(1 - r^+)$ is lower than the baseline profit share, $(1 - r)$. Given r^+, the value $W^j(h, x, p)$ of a type-x employment relationship with a worker of type j in segment (h, x) for a firm with productivity p is given by

$$W^j(h, x, p) = \underbrace{(1 - r^+)f(h, x, p)}_{\text{flow profit}} + \beta^f \big(1 - \delta^j(h, x, p)\big) W^j(h, x, p)$$

$$= \frac{(1 - r^+)f(h, x, p)}{1 - \beta^f\big(1 - \delta^j(h, x, p)\big)} \tag{16}$$

where β^f is the firms' discount factor. When posting a vacancy, the firm has to take the expectation over worker types as they differ in their on-the-job search effort which affects the separation probability and expected value of a match. The ex-ante expected value of filling a vacancy is thus

$$\mathbb{E}\big[W(h, x, p)\big] = \sum_j \frac{S^j(h, x, p)}{S(h, x, p)} W^j(h, x, p)$$

$$= (1 - r^+)f(h, x, p) \underbrace{\sum_j \frac{S^j(h, x, p)}{S(h, x, p)} \frac{1}{1 - \beta^f\big(1 - \delta^j(h, x, p)\big)}}_{\text{discounted expected match duration}} \tag{17}$$

Knowing the expected value of an employment relationship, the optimal number of vacancies has to satisfy

$$\kappa'(v, h, x) = \beta^f \eta(h, x, p) \mathbb{E}\big[W(h, x, p)\big] \tag{18}$$

Optimal vacancy posting then requires firms to post vacancies until the marginal cost of posting another vacancy is equal to the discounted expected value of an employment relationship weighted by the probability of filling the vacancy.

2.6 Equilibrium

A stationary equilibrium consists of value functions, $V_{lu}^j(h)$, $V_{su}^j(h, x, p)$, $V_e^j(h, x, p)$, search effort policy functions, $\ell^j(h, \sigma)$, vacancy posting policy functions, $v(h, x, p)$, labor market tightness, $\theta(h)$, distribution of vacancies, $\Psi(h, x, p)$, and a distribution of workers across states, $\Upsilon(j, h, \sigma)$,

that satisfy the following conditions. First, given labor market tightness and the distribution of vacancies, the value and search effort policy functions solve the workers' problem. Second, given labor market tightness, the distribution of vacancies, workers' search policies and the distribution of workers across states, firms' vacancy posting policy functions solve the firms' optimality conditions. Third, the distribution of workers across states is stationary. That is, given the economy starts at this distribution and given the policy functions and labor market tightness, the distribution of workers across states will not change.

3   Estimation

In this section, I first describe the pre-set parameters and parameterize workers' flow utility and skill distributions, firms' productivity distribution and vacancy posting cost function and the tax schedule (section 3.1). Second, I discuss which moments I target in the method of simulated moments in order to identify the jointly estimated parameters (section 3.2). Third, I evaluate the estimation results and model fit (section 3.4).

3.1   Parameterization and Pre-Set Parameters

One period in the model corresponds to one quarter. I set the quarterly discount factor of both workers and firms equal to $\beta = 0.98$ and choose the minimum wage of EUR 8.5 as the numéraire.

The employment level for full-time employment, e_f is normalized to one and e_p and e_m are set to match the ratio of average weekly hours of part-time and marginal workers relative to full-time employed workers reported by Dustmann et al. (2020) who have access to hours worked in the German security data. This yields $e_p = 0.615$ and $e_m = 0.223$.

I set $r_f = r_p = 0.62$ which approximately match the aggregate labor share in Germany between 2010 and 2014. The labor share for marginal jobs r_m is estimated and allowed to be lower in order to match the joint distribution of wages and employment levels. As marginal jobs constitute a tiny share of the aggregate wage bill, this does not affect the labor share significantly.

The German transfer system distinguishes between short- and long-term unemployment. During the first year of unemployment, workers are paid a fixed fraction $b = 0.6$ of their previous earnings (ALG I), but not less than the subsistence minimum B_{min}. With a constant net replacement rate for short-term unemployed workers, benefits differ by previous earnings. Long-term unemployed workers receive the subsistence minimum B_{min} independent of their previous earnings (ALG II). I set the policy parameter B_{min} to 800 EUR which corresponds to about 55% of of full-time monthly earnings at the minimum wage of 8.5 EUR. For employed workers, 80% of their net earnings is deducted from the amount of subsistence benefits they are eligible to receive on top of their earnings ($\tau_{top} = 0.8$). Hence, all workers with monthly net earnings of at least 1,000 EUR are not eligible for top-up transfers. Workers with net earnings below this threshold are eligible for subsistence transfers if they do not receive non-labor income y_{free}^j from their spouse. Using SOEP data that allow me to link spouses, I calculate average net earnings of the spouses of the married men and women in my sample. I then assign half of that amount to the spouse as non-labor income. On average, married women have roughly EUR

894 and married men EUR 409 in non-labor income from their spouses' net earnings. In the model, non-labor income is deducted from subsistence benefit eligibility. With $B_{min} = 800$, this implies that married women are not eligible for subsistence benefits and married men receive at most half of total subsistence benefits. Singles are assumed to have no non-labor income and are hence eligible for the full amount of subsistence benefits.

Gross earnings are subject to taxation. Note that I refer to the sum of taxes and social security contributions simply as taxes. I assume that workers pay a constant marginal tax rate τ^j on earnings above an exemption level D^j.

$$y_{net} = \min\{y_{gross}, D^j\} + (1 - \tau^j)\max\{0, y_{gross} - D^j\} \tag{19}$$

and estimate the parameters on SOEP data for gross and net earnings for the years 2013 and 2014 separately for different socioeconomic types. Figure 1 shows that the estimated average tax function provides a good fit to the binned data.

FIGURE 1: Fit of Estimated Tax Functions
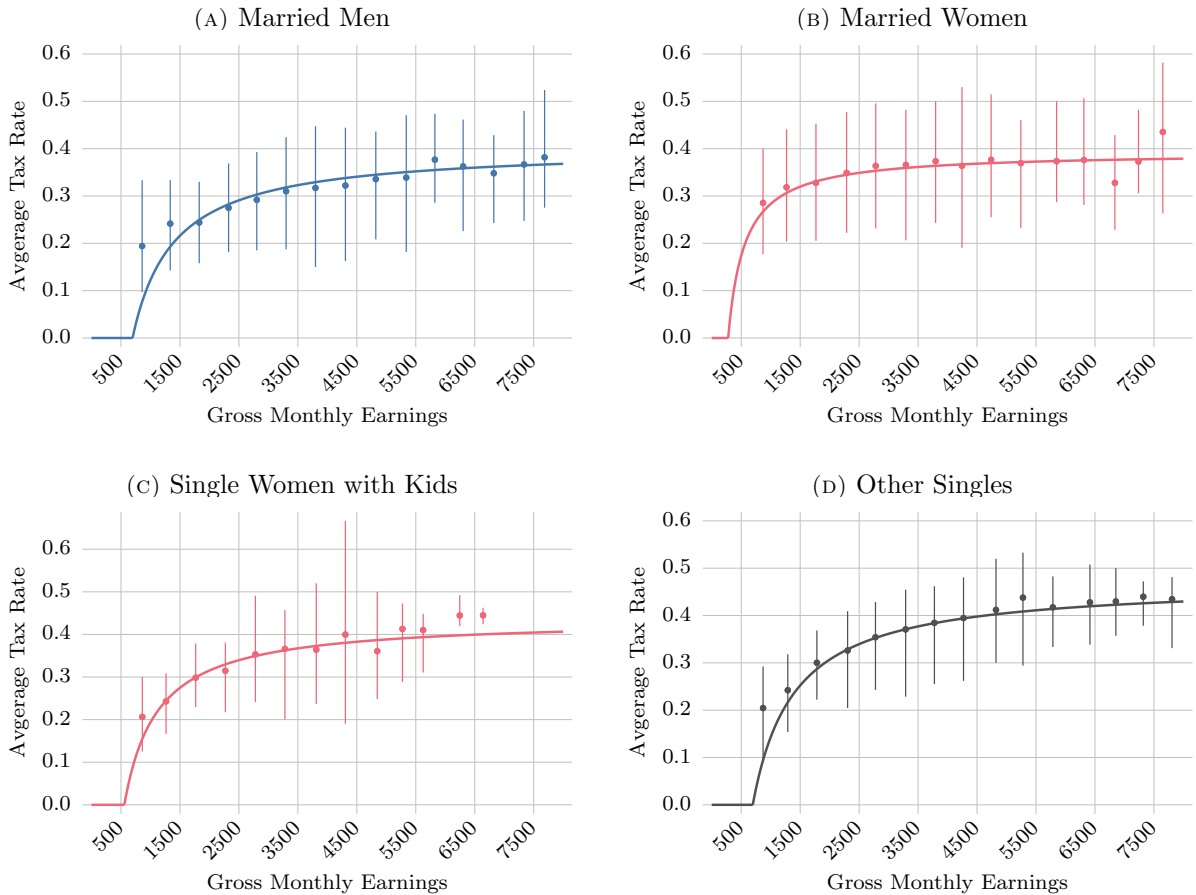
Note: This figure shows estimated average tax functions as well as the mean average tax rate in various gross earnings bins. The spikes show the range between the 10th and 90th percentile of average tax rates in those bins. The average tax function is $T(y) = (1 - \tau^j)\max\{0, y - D^j\}/y$.

I assume that firm productivity $p \geq 1$ is drawn from a Log Gamma distribution with shape and scale parameters α and θ. Productivity differences across job types are governed by $a_p, a_m \in (0, 1]$ with a_f normalized to one. Human capital is drawn from a gender-specific left-truncated

Log Normal distribution defined by μ_h^g and σ_h^g. The truncation bound h_{min} is chosen such that the lowest possible wage – resulting from a match between the least productive firm ($p_{min} = 1$) and lowest skilled worker generates a wage of 4 EUR, i.e. $rh_{min}p_{min}a_m = 4$. Data from the SOEP as well as the German Survey of Earnings Structure show that there are virtually no jobs with an hourly wage below 4 EUR (Minimum Wage Commission, 2018).

Workers' utility depends on consumption, job search and the employment level in the following way:

$$u^j(\ell|h,\sigma) = \frac{c^j(h,\sigma)^{1-\gamma_c}}{1-\gamma_c} - \ell^\zeta + h^\epsilon \sum_x \gamma_x^j \mathbb{1}\{x(\sigma) = x\} \tag{20}$$

where $\zeta^j > 1$ and γ_x^j are constants that capture the (dis-)like for the different employment levels (relative to nonemployment) for type-j workers. The state-specific constants will allow the model to match the distribution over employment levels for each sociodemographic group. The state-constants are scaled by h^ϵ where $\epsilon > 0$ implies that the absolute importance of the state-(dis-)utilities grows with human capital. The value of ϵ may helps to match the joint distribution of wages and employment levels.[19]

Finally, I assume that the cost of posting v vacancies for type-x jobs in skill segment h is given by

$$\kappa(v,h,x) = \underbrace{e_x \kappa_1}_{\equiv \kappa_{1x}} v^{\kappa_{2x}} f(h)^{1-\kappa_{2x}} \tag{21}$$

where f is the density of workers' human capital and e_x is the employment level.[20] The convexity of the cost function may depend on the job type. I scale the cost of posting vacancies by the density of human capital due to the assumption of segmented labor markets. This implies that optimal vacancy creation satisfies

$$v(h,p,x) = \left(\frac{(1-r^+)f(x,h,p)A(h,p,x)}{\kappa_{1x}\kappa_{2x}}\right)^{\frac{1}{\kappa_{2x}-1}} f(h) \tag{22}$$

where $A(h,x,p)$ is a term depending on the hiring probability and the discounted expected match duration. The elasticity of vacancy creation with respect to the profit share is $1/(\kappa_{2x} - 1)$.

3.2 Estimation Strategy

The remaining structural parameters will be estimated using the simulated method of moments to match important aspects of the German labor market in 2014. I estimate the model using a two-step multiple-restart procedure similar to the TikTak-estimation method proposed by Arnoud et al. (2019). In the first stage, I search a compact parameter space by evaluating the objective function at about three million quasi-random Sobol points. I then select the best three

---

[19]For example, if flow utility of consumption is linear $\gamma_c = 0$, $\gamma_p > \gamma_f$ and $\epsilon = 0$, the surplus of part-time work over full-time work will be larger smaller for high-skill workers compared to low-skill workers resulting in relatively more part-time jobs in the lower skill segments.

[20]This functional form is similar to those used in Shephard (2017) and Engbom and Moser (2018).

thousand points as starting points for local minimizations and pick the local minimizer with the lowest local minimum as the global minimizer.

The parameters to be jointly estimated are the gender-specific skill distribution parameters (α^g, θ^g), the firm productivity distribution parameters (μ_p, σ_p), the sociodemographic-specific preference parameters (γ_x^j), the type-independent preference parameters $(\gamma_c, \zeta, \varepsilon)$, the search efficiency parameters $(\phi^{su}, \phi^{lu}, \phi^{e_x})$, the vacancy cost parameters (κ_1, κ_{2x}), the mass of firms (m_f), the probability of becoming long-term unemployed $(\pi_{lu|su})$, and the labor share of marginal jobs (r_m).

To inform these parameters, I target (a) the joint distribution of labor market states and sociodemographics, (b) average and sociodemographic-specific job finding rates out of unemployment, (c) the average elasticity of job finding probabilities with respect to unemployment insurance for short-term unemployed workers, (d) job-to-job transition probabilities conditional on employment level, (e) selected wage quantiles conditional on gender and employment level, (f) the distribution of gender and employment levels in selected wage groups, (g) selected quantile ratios of the gender-specific distributions of worker fixed effects of full-time workers, (h) selected quantile ratios of the distribution of full-time clustered firm fixed effects weighted by the number workers in each employment level, (i) the standard deviation of the log of full-time firm size, and (j) the aggregate job vacancy rate.

While all of the parameters are jointly identified by all moments, I will provide intuition for the selection of moments. In addition, appendix D exploits the multiple restart design to illustrate ex-post which parameters are informed by which moments.

In the absence of a minimum wage, the wage equation in my model is very simple. As in Abowd et al. (1999) (henceforth AKM), the wage w of a full-time worker employed at firm with productivity p is log-additive in her skill h and the firm's productivity

$$\log(w) = \log(r) + \log(h) + \log(p) \tag{23}$$

where r is the exogenous piece-rate. I estimate the empirical distribution of worker and firm-class fixed effects using a clustered AKM approach (Bonhomme et al., 2019). In particular, I first cluster firms based on their wage distributions and use firm-class fixed effects instead of firm fixed effects. See Appendix B for details.

To inform the parameters of the skill and productivity distributions, I target selected quantile ratios of the distribution of worker (by gender) and firm fixed effects for full-time workers as well as selected quantile ratios of the distribution of full-time firm fixed effects weighted by the number of part-time and marginal jobs.

Apart from the fixed effects distributions, I target selected quantiles of the gender-specific wage distributions and the overall wage distributions of full-time, part-time and marginal workers. Explicitly targeting the wage distribution is important as the model needs to be able to replicate the pre-reform distribution of wages and employment levels as well as possible.

The search efficiency parameters are closely related to the average job finding probability of short- and long-term unemployment as well as the probability of job-to-job transitions conditional on the current employment level.

The (dis-)utility parameters γ_f^j, γ_p^j and γ_m^j drive heterogeneity in employment status across sociodemographics. The curvature-parameter ζ in the disutilty of job search affects the elasticity of job search with respect to the surplus of employment. Based on the quasi-experimental literature on the UI-elasticity of job finding probabilities I target an average elasticity of 0.5 across all workers (e.g. Chetty, 2008; Schmieder et al., 2012).

The scale parameter κ_1 affects the overall labor market tightness by making vacancies more or less costly and is thus related to the job vacancy rate. The curvature parameters κ_{2x} affect the share of type-x jobs across skill-segments and hence across the wage distribution. Increasing κ_{2m} relative to κ_{2f} will lead to more type-x vacancies in low skill segments as type-x vacancy posting becomes more inelastic with respect to the expected value of vacancy which in turn tends to increase in h. Moreover, decreasing κ_{2f} will make it easier for more productive firms to grow large relative to unproductive firms such that the standard deviation of the log of full-time firm size increases. The curvature parameters are thus informed by both the share of part-time and marginal jobs across the wage distribution as well as the standard deviation of the log of full-time firm size.

3.3 Data

The main data source is a 2% sample of administrative social security records of German workers (SIAB) from 2011 to 2014. The SIAB is a linked employer-employee data set containing information on daily earnings and employment levels (full-time, part-time and mini-job). Sociodemographic characteristics (apart from gender and age) are only available for nonemployed workers. I thus complement it with survey data from the German Socioeconomic Panel (SOEP) which contains annual information on more than 15 thousand workers. For firm-level moments I use administrative data from the Establishment History Panel and the Job Vacancy Survey of the Institute for Employment Research (IAB). I focus on prime-aged workers aged 25 to 60.

3.4 Estimation Results

The model parameters are reported in Table 2 and 3. The skill distribution of men has a higher mean but lower standard deviation than that of women. Figure 2 show the distributions of human capital and firm productivity.

Table 2 shows that, apart from married men, workers receive utility from working fewer hours as $\gamma_f^j < \gamma_p^j < \gamma_m^j$. All women have a higher preference for part-time and marginal jobs. Single women with kids receive the highest disutility from working full-time. The convexity of search cost is close to two. The positive value for ϵ implies that the state (dis-)utilities are scaled up in higher skill segments.

Table 3 shows the firm and labor market parameters. The within-firm relative productivity of part-time and marginal jobs is estimated to be 1.05 and 0.91 respectively.

The vacancy posting cost function for full- and part-time jobs is not very convex as $\kappa_{2f} = 1.75$, $\kappa_{2p} = 1.53$ and $\kappa_{2m} = 2.09$ are not substantially greater than two.[21]

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[21]For Brazil, Engbom and Moser (2018) estimate a value of 1.45. Shephard (2017) assumes a quadratic vacancy posting cost function in the UK.

Name	Description	Value	Source
All Workers			
β	Discount factor	0.980	–
γ_c	CRRA parameter	0.727	estimated
ζ_2	Search disutility (convexity)	2.056	estimated
ϵ	Relation btw. h and state utilities	0.173	estimated
Skill Distribution of Men			
μ	Mean of $\log(h)$	2.920	estimated
σ	Std. dev. of $\log(h)$	0.542	estimated
Skill Distribution of Women			
μ	Mean of $\log(h)$	2.725	estimated
σ	Std. dev. of $\log(h)$	0.517	estimated
Men, Single			
γ_f^j	State utility of $s = f$	-0.070	estimated
γ_p^j	State utility of $s = p$	-0.117	estimated
γ_m^j	State utility of $s = m$	0.484	estimated
Men, Married			
γ_f^j	State utility of $s = f$	0.384	estimated
γ_p^j	State utility of $s = p$	0.130	estimated
γ_m^j	State utility of $s = m$	0.480	estimated
Women, Single, No Kids			
γ_f^j	State utility of $s = f$	0.007	estimated
γ_p^j	State utility of $s = p$	0.226	estimated
γ_m^j	State utility of $s = m$	0.857	estimated
Women, Single, Kids			
γ_f^j	State utility of $s = f$	-0.501	estimated
γ_p^j	State utility of $s = p$	0.531	estimated
γ_m^j	State utility of $s = m$	0.896	estimated
Women, Married			
γ_f^j	State utility of $s = f$	-0.210	estimated
γ_p^j	State utility of $s = p$	0.984	estimated
γ_m^j	State utility of $s = m$	1.962	estimated

The top bars in each of the panels of Figure 3 show that the model is able to capture the overall distribution of labor market states and job finding rates.[22] In the estimated model (data), 7.5% (6.4%) of workers are unemployed with 51.4% (51.8%) of them in long-term unemployment. Among the employed workers, 9.0% (9.6%) have a marginal job, 27.4% (24.0%) work part-time and 63.6% (66.3%) have a full-time job. The job finding rate out of short-term unemployment is 28.5% (29.6%) and considerably lower for long-term unemployed workers with 7.0% (6.7%). The difference in job-finding rates reflects the fact that search is estimated to be substantially less efficient in generating matches with firms ($\phi_{lu} < \phi_{su}$). In addition, long-term unemployed workers have lower human capital and thus lower incentives to search for jobs compared to short-term unemployed workers.

The estimated model is also able to capture most of the heterogeneity across sociodemographic groups. Compared to men, a much larger share of women and in particular single women with kids and married women work in part-time or marginal jobs. While the model can

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[22]See table A.3 for the values underlying Figure 3.

TABLE 3: Firm, Labor Market and Policy Parameters

| Name	Description	Value	Source
Firms			
m	Mass of firms	0.025	estimated
α	Scale of $log(p)$	2.269	estimated
θ	Shape of $log(p)$	0.106	estimated
α_f	Relative productivity $(x = f)$	1.00	normalized
α_p	Relative productivity $(x = p)$	1.05	estimated
α_m	Relative productivity $(x = m)$	0.91	estimated
κ_{1f}	Vacancy posting cost (weight), $x = f$	100.0	estimated
κ_{1p}/κ_{1f}	Relative vacancy posting cost, $x = p$	0.850	estimated
κ_{1m}/κ_{1f}	Relative vacancy posting cost, $x = m$	0.791	estimated
κ_{2f}	Vacancy posting cost (convexity), $x = f$	1.750	estimated
κ_{2p}	Vacancy posting cost (convexity), $x = p$	1.534	estimated
κ_{2m}	Vacancy posting cost (convexity), $x = m$	2.087	estimated
Labor Market			
ξ	Vacancy-elasticity of matches	0.3	literature
$\bar{r}_F$	Wage rate $(x = f)$	0.605	???
$\bar{r}_p$	Wage rate $(x = p)$	0.605	???
$\bar{r}_x$	Wage rate $(x = m)$	0.548	???
e_f	Hours $(x = f)$	1.0	normalized
e_p	Hours $(x = p)$	0.615	SOEP
e_m	Hours $(x = m)$	0.223	SOEP
$\pi_{su|e_f}$	Transition from e_f to su	0.010	SIAB
$\pi_{su|e_p}$	Transition from e_p to su	0.019	SIAB
$\pi_{su|e_m}$	Transition from e_m to su	0.030	SIAB
$\pi_{lu|su}$	Transition from su to lu	0.075	estimated
ϕ_{su}	Search efficiency, $s = su$	0.337	estimated
ϕ_{lu}/ϕ_{su}	Relative search efficiency, $s = lu$	0.384	estimated
ϕ_{e_f}/ϕ_{su}	Relative search efficiency, $s = e_f$	1.147	estimated
ϕ_{e_p}/ϕ_{su}	Relative search efficiency, $s = e_p$	0.911	estimated
ϕ_{e_m}/ϕ_{su}	Relative search efficiency, $s = e_m$	0.834	estimated
ψ_f	Godfather shock, $x = f$	0.017	SIAB
ψ_p	Godfather shock, $x = p$	0.022	SIAB
ψ_m	Godfather shock, $x = m$	0.050	SIAB

replicate the observed heterogeneity in employment levels, the unemployment rate of single men and especially single women with kids and married women is less than perfectly matched.

Figure 4 and table A.6 show the distribution wages over selected wage bins. The overall fit (panel A) is remarkably good given the limited flexibility imposed by the parametric skill and productivity distributions and the fact that there are no skill-dependent parameters.[23] Only 2.4% (1.8% in the data) of all jobs pay a wage below 6.5 Euro, 8.5% (9.8%) of wages are above 6.5 Euro but below 8.5 Euro, 22.1% (18.8%) of wages are between 8.5 and 12.5 Euro, 33.6% (34.6%) are between 12.5 and 20 Euro and 33.4% (35.0%) of wages exceed 20 Euro. The model is also able to capture gender-specific heterogeneity as a larger share of women find themselves in the lower wage bins. Similar to the data, 14.1% (16.5%) of women are affected by the initial minimum wage, only 7.8% (6.7%) of men earn less than 8.5 Euro per hour. However, the right tail of the wage distribution of men is too short while that of women is too long relative to the data.

The differences in the job-type-specific wage distribution distribution (panels B to D) are also replicated by the model. Full-time jobs pay substantially higher wages than part-time jobs

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FIGURE 2: Estimated Human Capital and Firm Productivity Distributions
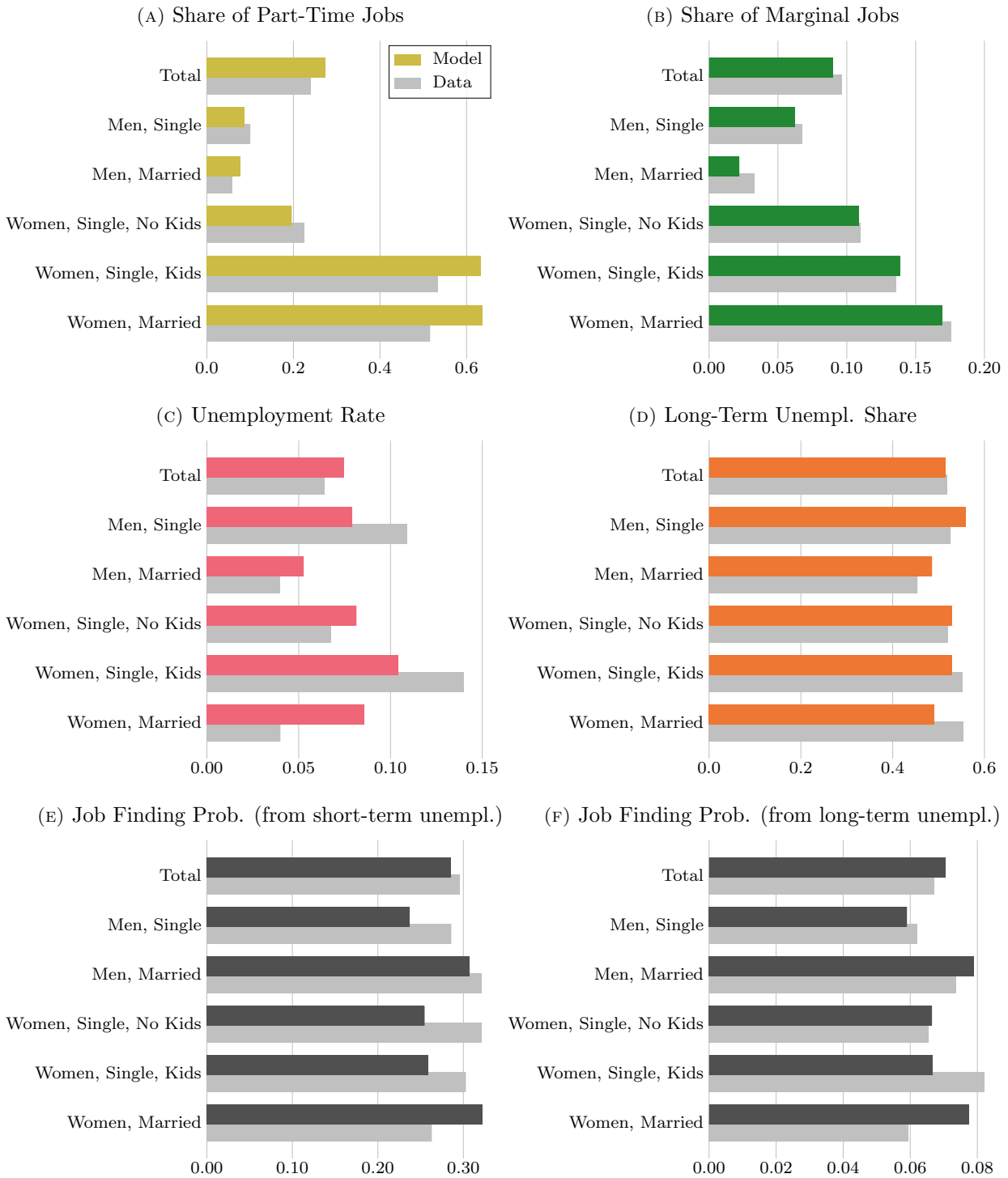

(A) Workers

(B) Firms
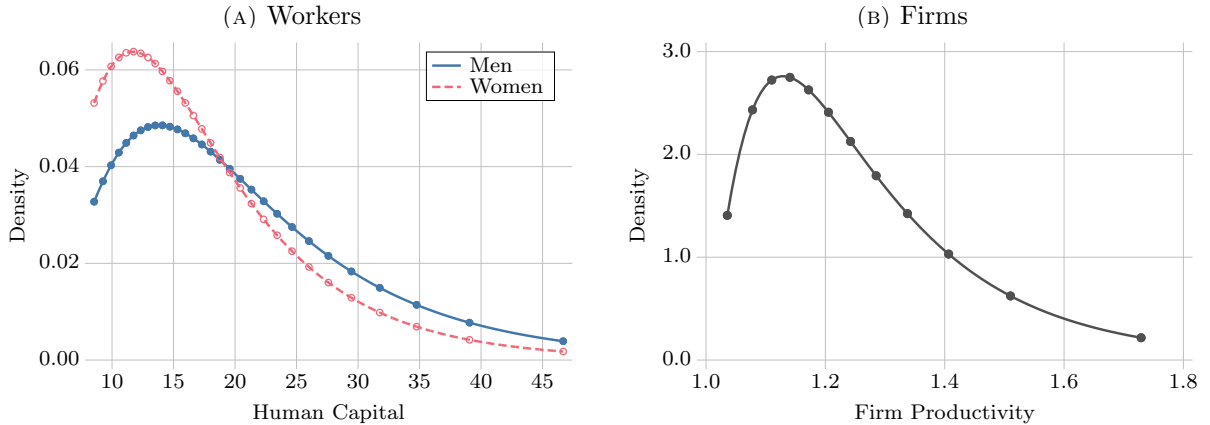
Note: This figure shows the density of the estimated human capital distributions of workers (by gender) and firm productivity distribution. All distributions are truncated log normal distributions. The markers refer to the grid points used to discretize the distribution.

which in turn pay higher wages than marginal jobs. Hence, minimum wages will cut deeper into the wage distribution of part-time and marginal jobs compared to full-time jobs. In particular, the initial minimum wage affects 45.8% (53.9%) of marginal jobs, 10.8% (12.1%) of part-time jobs but only 5.8% (5.5%) of full-time jobs. The most important difference between model and data is that the distribution of wages for marginal jobs is too dispersed. There are too many jobs paying a wage below 6.5 Euro or above 12.5 Euro and too few jobs in the range between 6.5 and 12.5 Euro. In addition, too few full-time jobs pay wages between 8.5 and 12.5 Euro. This will affect how the distribution of job types is affected by the minimum wage. Figure 5 shows the share of full-time, part-time, marginal jobs and men in each of these wage bins. Marginal jobs are over-represented in the lowest wage bin. In addition, part-time jobs are over-represented in the wage bins around the initial minimum wage of 8.5 Euro as there are not enough full-time jobs in this range. While these differences between model and data need to be kept in mind, the model delivers a good fit to the joint distribution of wages and job types which is a complicated object.

The distribution of worker and firm fixed effects for full-time jobs is shown in Figure 6. Figure 7 shows the distribution of full-time firm fixed effects among part-time and marginal jobs. In particular, panels C and D show the employment weighted variation in firm productivity among part-time and marginal jobs which the model is able to match quite closely. Panels E shows the percent difference between the qth quantile of the firm productivity distribution weighted by part-time employment and the corresponding quantile of the firm productivity distribution weighted by full-time employment. Both in the data and the model, firm productivity is just slightly lower among full-time workers (about 5%). Using marginal workers as weights instead of full-time workers, the firm productivity distribution shifts downward by around 20% in the data but by significantly more in the model (panel F). Hence, marginal workers in the model work at firms that pay too low full-time wages compared to the data.[24] Table 4 shows the variance decomposition of full-time wages. Worker heterogeneity contributes 83.9% (54.4%),

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[24]See Table A.7 and Table A.8 for details.

FIGURE 3: Model Fit – Employment Moments

(A) Share of Part-Time Jobs

(B) Share of Marginal Jobs

(C) Unemployment Rate

(D) Long-Term Unempl. Share

(E) Job Finding Prob. (from short-term unempl.)

(F) Job Finding Prob. (from long-term unempl.)

Note: This figure shows labor market moments targeted in the estimation for the full population (Total) and within the sociodemographic groups. Subfigures 1 and 2 show the probability of working a part-time and marginal job conditional on being employed. Subfigure 3 shows the unemployment rate and subfigure 4 the share of long-term unemployed workers conditional on being unemployed. Figures 5 and 6 show the job finding probabilities for short- and long-term unemployed workers. Data: SIAB, SOEP.

FIGURE 4: Model Fit – Wage Groups by Job Types and Gender
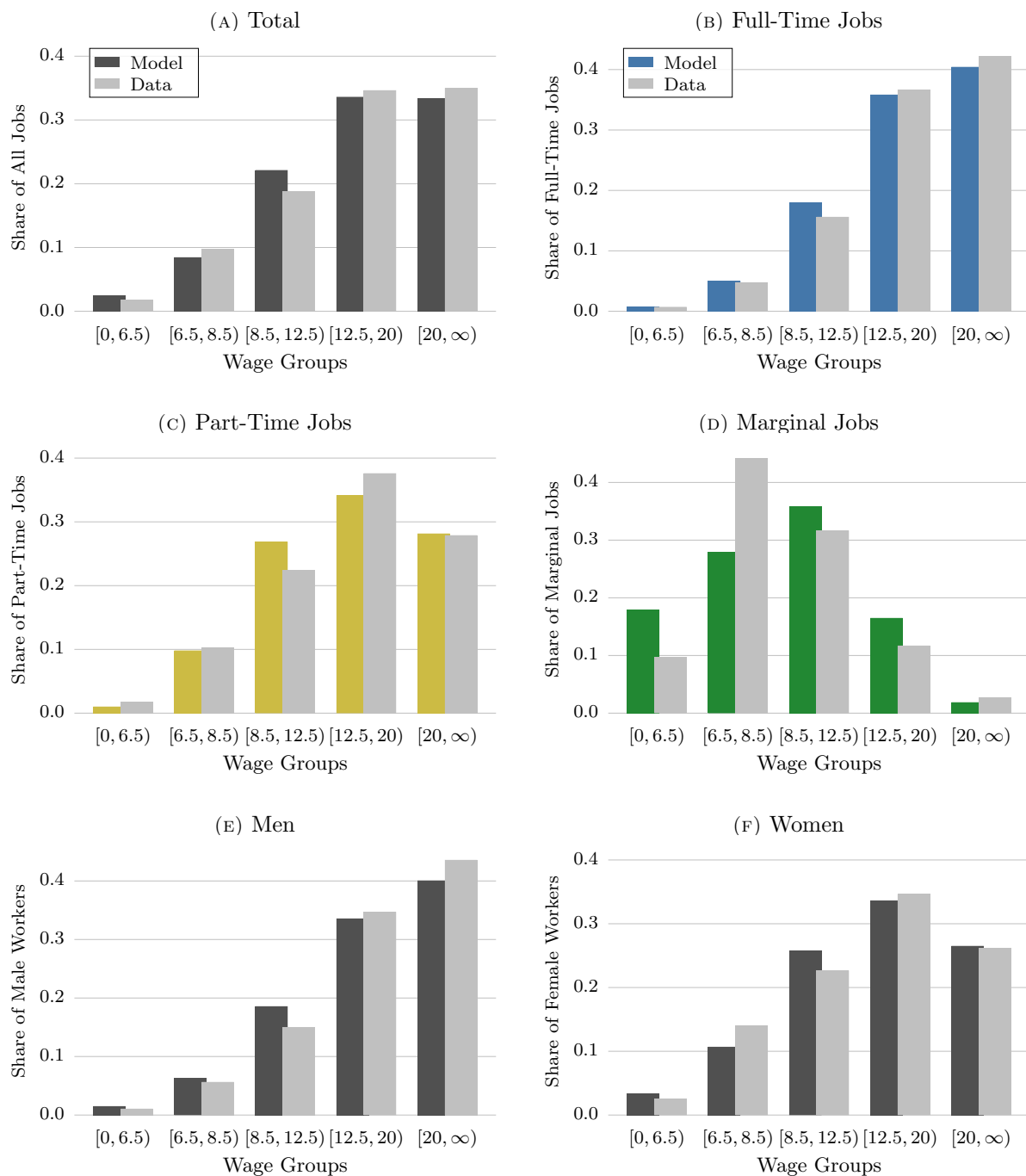
Note: This figure shows the distribution of jobs over four wage groups for all workers and separately for full-time, part-time, marginal job, male and female workers in the model and data. Data: SIAB, SOEP.

FIGURE 5: Model Fit – Job Types and Gender By Wage Groups

(A) Full-Time Jobs

(B) Part-Time Jobs

(C) Marginal Jobs

(D) Men
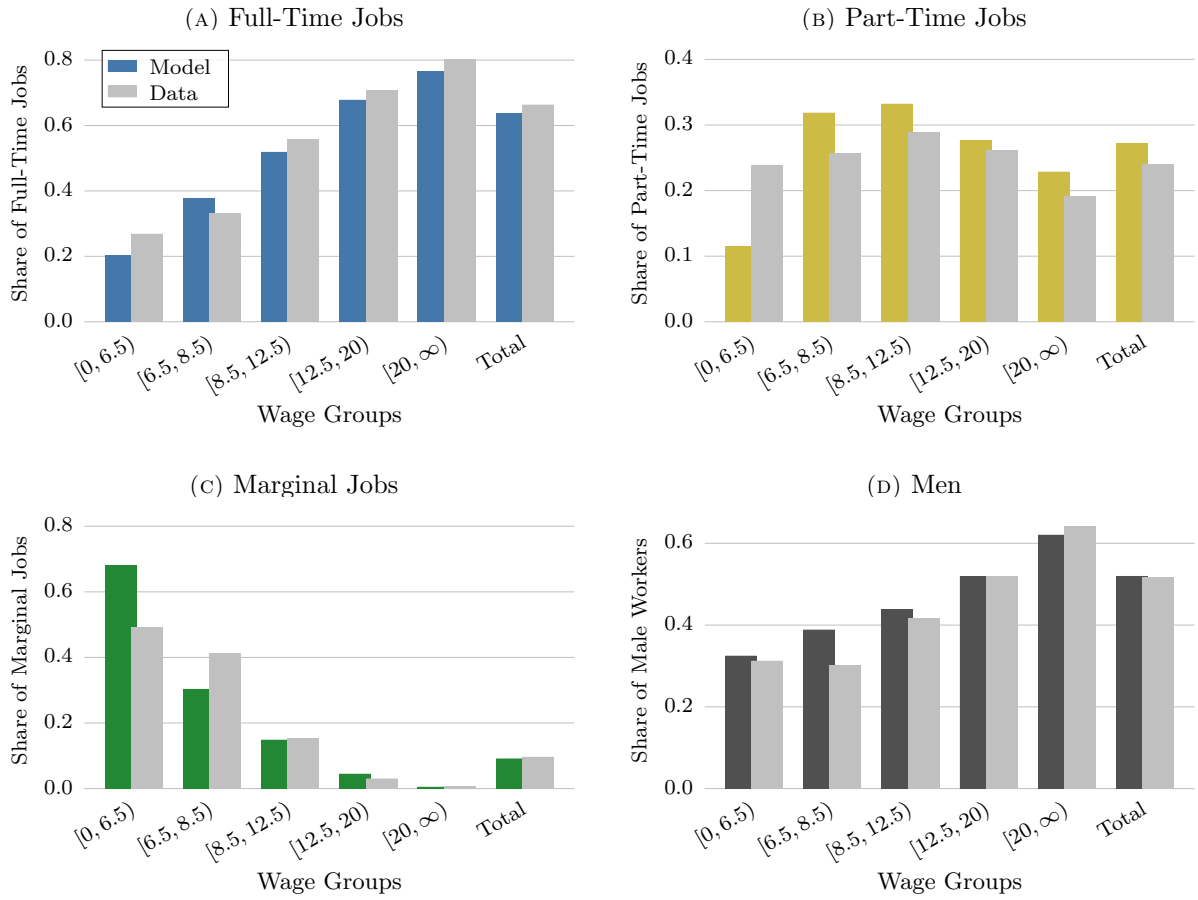
Note: This figure shows the share of full-time, part-time and marginal jobs as well as the share of men within various bins of the wage distribution in the model and data. Data: SIAB, SOEP.

FIGURE 6: Model Fit – Clusterd AKM Fixed Effects

(A) Men
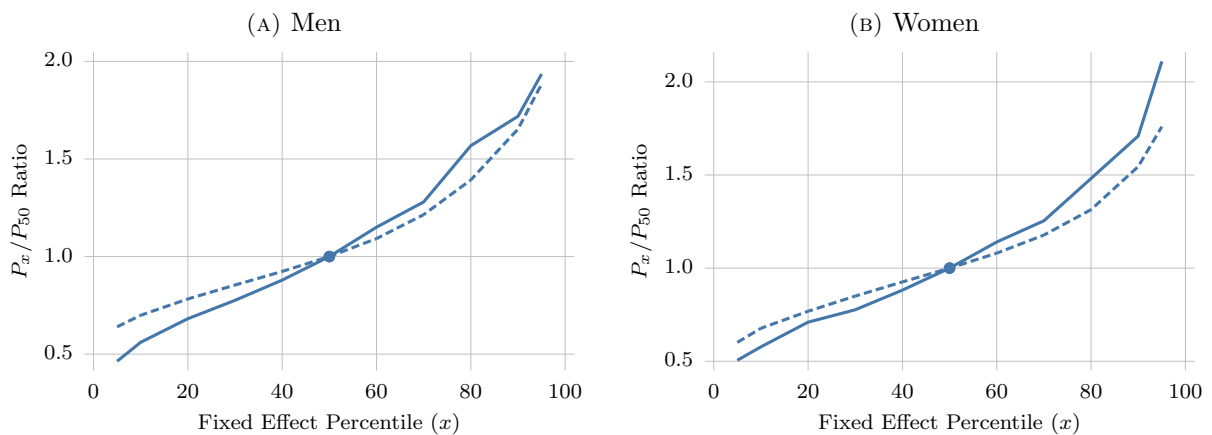
(B) Women

Note: This figure shows the ratios of selected percentiles to the median of the distributions clustered AKM worker fixed effects for men and women. See appendix B for details. Data: SIAB.

FIGURE 7: Model Fit – Firm Fixed Effect
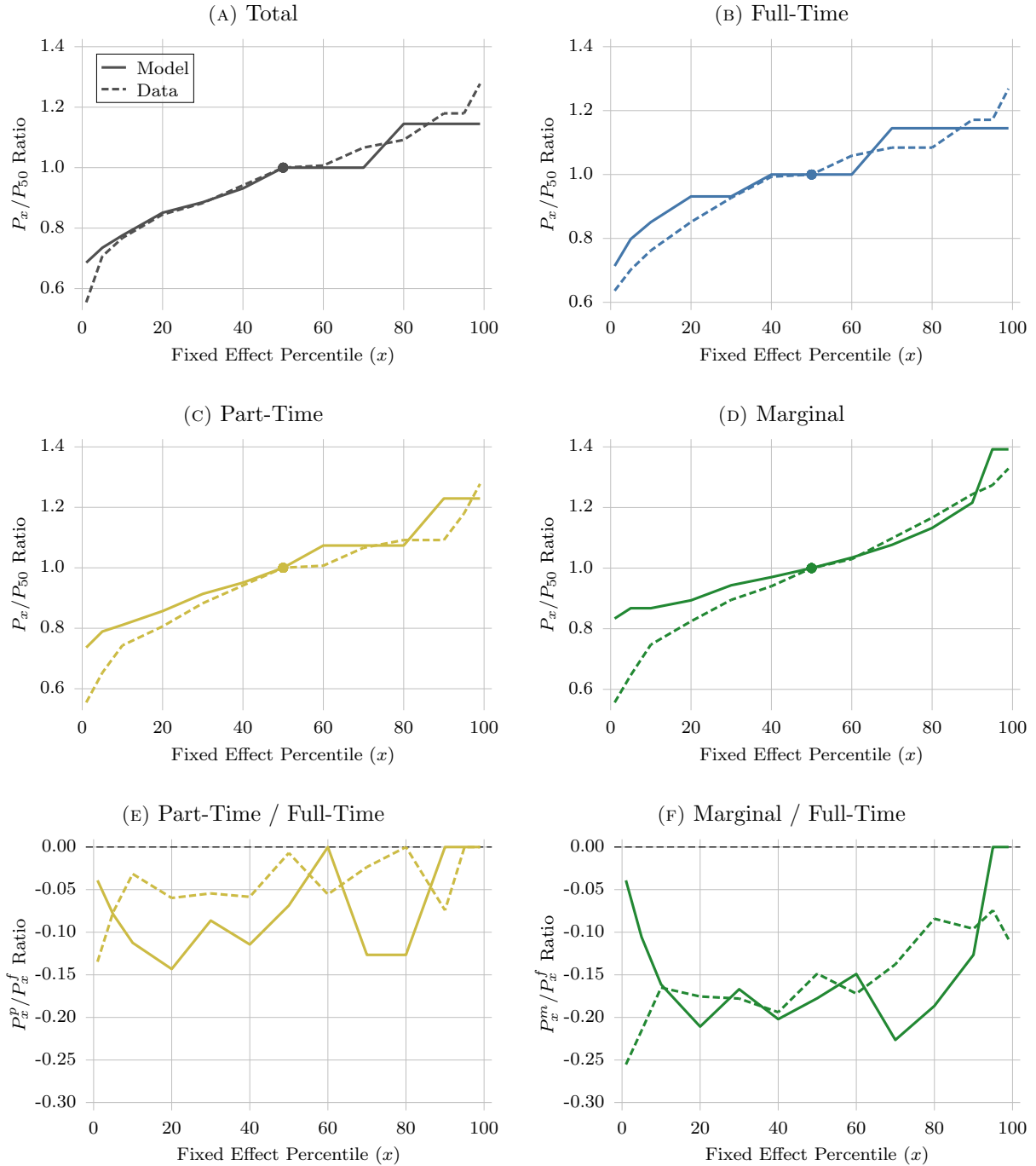
Note: This figure shows the distribution of (clustered) firm fixed effects estimated using clustered AKM on full-time jobs. In panels A, B, C and D, all jobs, only full-time, only part-time jobs and only marginal jobs are used as weights respectively. Panels E and F show how the distributions change when weighting by part-time and marginal jobs instead of full-time jobs. Data: SIAB.

TABLE 4: Model Fit – Clustered AKM Wage Decomposition

	Total $var(\ln w)$	Workers $var(\ln h)$	Firms $var(\ln p)$	Sorting $2cov(\ln h, \ln p)$	$corr(\ln h, \ln p)$
Value					
Data	0.219	0.119	0.028	0.072	0.624
Model	0.213	0.175	0.016	0.022	0.215
Share					
Data	–	54.4 %	12.8 %	32.9 %	–
Model	–	82.2 %	7.3 %	10.5 %	–

Note: This table shows the variance decomposition of log wages into a worker component, a firm component and their covariance. The worker and firm-class fixed effects are estimated using the years 2011 to 2014 and 25 firm classes. The column "Total" refers to the total *explained* variance, i.e. the total variance minus the residual variance. In the data, the residual variance accounts for only 3% of the total variation. In the model, there is no distinction between explained and total variance. Data: SIAB, own calculations.

firm heterogeneity 6.1% (12.8%) and sorting 10.0% (32.9%) to the overall variation in full-time wages. The correlation between worker and firm fixed effects is 0.221 in the model and 0.624 in the data.[25] The fact that the model cannot capture this large degree of positive sorting implies that I underestimate the overall variation in full-time wages by about four log points. The fact that the model cannot capture this large degree of positive sorting implies that I underestimate the overall variation in full-time wages by about four log points.

4 The German Minimum Wage Reform of 2015

In 2015, the German government introduced a federal minimum wage of 8.5 EUR (Kaitz index of 47%) that cut deep into the wage distribution affecting more than 10% of all jobs. In this section, I use the estimated model to analyze how the initial federal minimum wage affected employment, productivity and output. First, I compare the pre- and post-reform steady states and highlight the mechanisms at play (4.1). Second, I analyze the transitional dynamics (4.4).

4.1 Steady State Comparison

The unemployment rate decreases slightly by 0.035 percentage points. There are two reasons for this muted employment effect. First, the minimum wage increases the surplus of employment and thus the incentive to search for a job which prevents job finding rates from falling too far. Column 4 shows that if firms' vacancy posting is held fix, job finding rates increase substantially and the unemployment rate drops by 0.09 percentage points. In contrast, when workers' search effort is held constant and only firms' vacancy posting decisions adjust, the jobfinding rate drops more strongly and the unemployment rate goes up by 0.01 percentage points.

Second, the decrease in the job finding rate out of unemployment is counterbalanced by a decrease in the job destruction probability of 0.02 percentage points. The minimum wage reallocates workers away from marginal towards part-time and full-time jobs. As these jobs

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[25]The correlation of 0.624 in the data is rather high. Using the same methodology, Bonhomme et al. (2019) find a correlation of 0.5 for Sweden. In order to match the observed correlation of worker and firm fixed effects, one may extend the model to make the probability of job destruction dependent on worker skill and firm productivity (true in the data).

have lower job destruction rates, the average job destruction probability drops. In particular, the share of marginal jobs among all jobs drops from 9.14% to 7.94%. The share of part-time and full-time jobs increases by 0.81 and 0.39 percentage points respectively. This shift in the distribution of job types is influenced by both workers' and firms' decisions. The minimum wage raises the surplus of working longer hours and induces marginal workers to search more intensely for part-time or full-time jobs. However, comparing columns 4 and 5 reveals that firms' vacancy posting decisions account for the majority of the shift towards jobs with more hours.

Average wages in the new stationary equilibrium are up by about 2.1%. Part of this increase is driven by reallocation to more productive firms. In other words, workers now work at firms where they would have received 0.5% higher wages in the absence of a minimum wage. While over two thirds of the increase in productivity reflects reallocation to more productive firms (higher p), part of the increase in productivity ($a_x p$) is a direct result of the shift away from relatively unproductive marginal jobs. Note that this is broadly consistent with the evidence reported by Dustmann et al. (2020) who show that about 25% of the wage increase of employed workers can be attributed to the reallocation channel.

Average gross earnings increase by more than wages (+3.5%) reflecting the shift towards jobs with longer hours (+1.4%). Taxes and transfers result in a 2.8% increase in average earnings and a 0.8% increase in incomes. The relatively weak increase in incomes follows from the fact that many low-skill workers top up their earnings with unemployment benefits. Reallocation to better firms and longer hours leads total output to grow by 0.5%. While the tax-and-transfer scheme mutes the increase in incomes, total transfer payments decrease by 6.0%. In addition, the government's revenues from labor taxation increase by 0.9% as average earnings grow and the unemployment rate falls slightly.

In sum, the minimum wage moves the economy into an equilibrium with higher productivity, output and employment. While the unemployment rate decreases only slightly, employment weighted by hours worked increases markedly as the share of part-time and full-time jobs rises. While the tax- and especially the transfer-system prevents incomes from growing more strongly, workers are less reliant on government transfers to top up their earnings. Combined with the fact that higher average earnings raise tax revenues, the reform improved the government's budget position.

4.2 Mechanisms

Figure 8 compares the effect on unemployment, output, hours worked and firm productivity in different partial equilibrium scenarios to the general equilibrium effects. In order to assess the importance of workers' search effort, the surplus of successful search, firms' vacancy posting, I switch off these channels one at a time.[26]

As expected, eliminating workers' search effort pushes the unemployment rate up while shutting down firms' vacancy posting pushes it down. Taking a closer look at the role of vacancy posting, we see that there are two opposing effects. On the one hand, the total mass of vacancies is reduced which drives up unemployment (via lower job finding rates). On the other hand, the change in the composition of posted vacancies away from unstable low-hours

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[26]See Table A.10 for details.

TABLE 5: Minimum Wage Effects – General Equilibrium

|	(1)	(2)	(3)
	Baseline ($\bar{w} = 0$)	New Equilibrium ($\bar{w} = 8.5$)	
	Value	Value	Change
Labor Market States			
Unemployment Rate	7.44%	7.40%	-0.035
Long-Term Share	51.17%	51.34%	0.170
Full-Time Share	63.60%	63.99%	0.394
Part-Time Share	27.26%	28.07%	0.812
Marginal Share	9.14%	7.94%	-1.206
Transition Probabilities			
$\Pr(e|u)$	17.54%	17.42%	-0.124
$\Pr(su|e)$	1.41%	1.39%	-0.017
Wages, Earnings & Incomes			
Log Wages	2.776	2.796	0.021
Log Productivity	0.382	0.388	0.005
Log Hours	3.389	3.403	0.014
Log Earnings	7.631	7.665	0.035
Log Net Earnings	7.279	7.308	0.028
Log Income	7.583	7.590	0.008
Macro Aggregates			
Log Output	8.305	8.310	0.005
Log Transfers	4.554	4.493	-0.060
Log Labor Taxes	6.719	6.728	0.009

Note: This table shows the long-run general equilibrium effects of the introduction of a federal minimum wage of 8.5 EUR relative to the baseline equilibrium without a minimum wage (first column). Changes refer to the absolute difference to the baseline outcome (e.g. percentage points or log points).

jobs lowers unemployment (by reducing the average job destruction rate). Besides this effect on average job destruction rates, the change in the hours-distribution of vacancies raises searchers' expected disutility from longer working hours and thus dampens the increase in the surplus of successful search and hence search effort and job finding rates. The reduction in overall vacancy posting, however, dominates such that the net effect of endogenous vacancy posting drives up unemployment.

Just as the minimum wage lowers firms' surplus of finding a worker, it raises workers' surplus of finding a firm. Endogenous search effort can thus offset the firm effect on unemployment. The increase in the surplus successful search increases workers' search effort. This directly pushes up job finding rates but also lowers labor market tightness which raises the contact rate for firms. Fixing only the surplus pushes unemployment up by much more than just fixing search effort all together. This is because lower vacancy creation and labor market tightness reduce the effectiveness of search (see equations 4 and 12).

The increase in average hours worked and firm productivity is driven by firms' vacancy posting and in particular by the change in the composition of vacancies. In general, firms create fewer vacancies for jobs that are (strongly) affected by the minimum wage. As the minimum wage affects low-hours and low-productivity jobs relatively often, the reduction in vacancies is not symmetric across employment levels. Conditional on meeting a firm, the probability of

FIGURE 8: Mechanisms - Partial vs. General Equilibrium

(A) Unemployment Rate

(B) Output

(C) Average Hours Worked

(D) Average Productivity
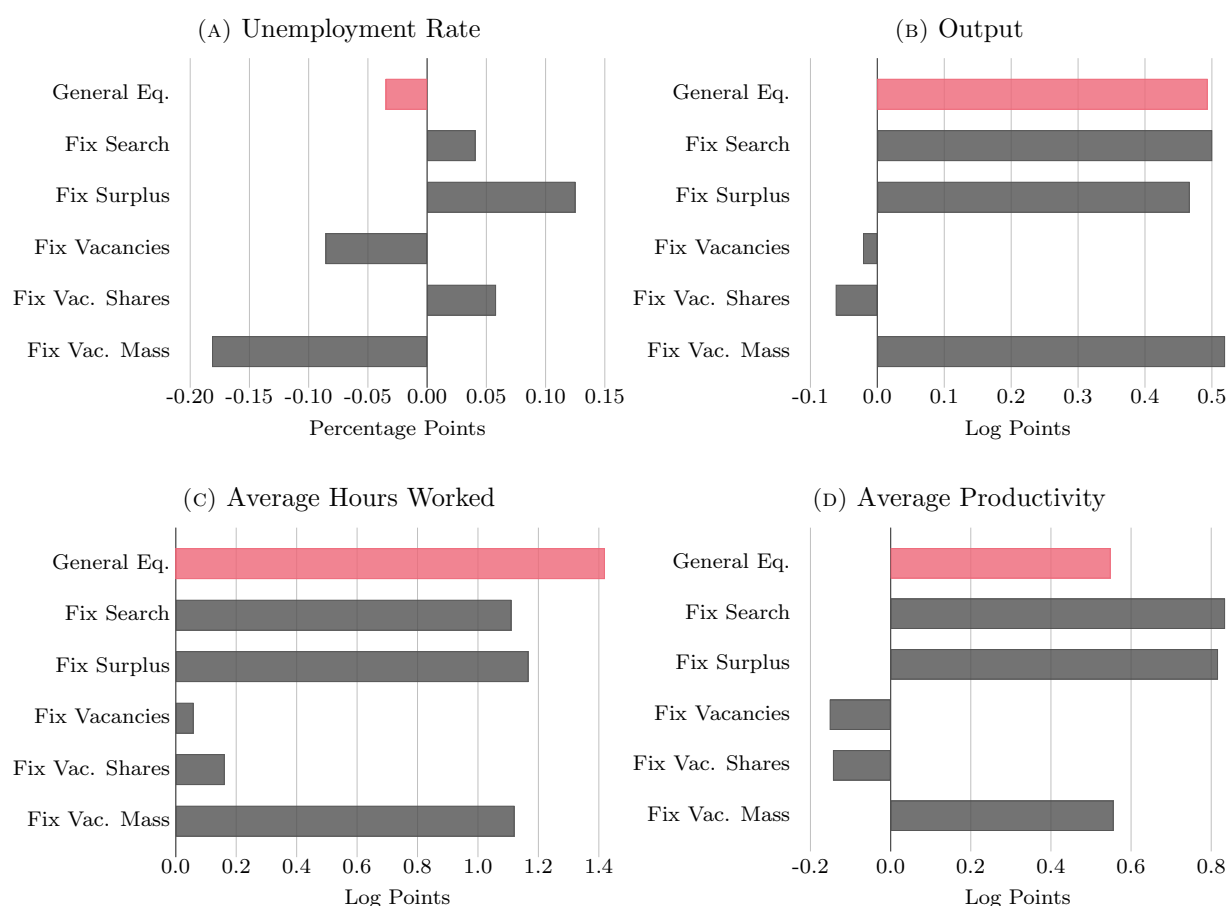
Note: This figure shows the effects of the minimum wage of 8.5 EUR for different partial equilibrium scenarios that result from shutting down different margins of adjustment. If a partial equilibrium differs from the general equilibrium scenario, the respective channel is important for generating the general equilibrium effect.

being offered a low-hours or low-productivity job declines. Figure 9 shows how the productivity distribution of vacancies changes.

4.3 Heterogeneity Across Sociodemographics

The different sociodemographic groups in the model and the data are differently affected by the minimum wage. Figure 10 shows that women are significantly more likely to earn less than 8.5 EUR per hour. I now analyze how the effects of the reform vary across sociodemographic characteristics in the new stationary equilibrium.

Panel B of Figure 10 displays the percentage point changes in the distribution of labor market states (full-time, part-time, marginal and total employment). All bars sum to zero. While the reallocation pattern away from marginals towards part-time and full-time jobs is the same qualitatively, there is substantial variation in magnitude. en and single women without kids move to both part-time and full-time jobs. In contrast, the share of married women and single women wit kids working full-time jobs hardly increases because of the high disutility of working full-time for this group. As a result, their unemployment rate increases slightly while total unemployment drops.

FIGURE 9: Productivity Distribution of Vacancies
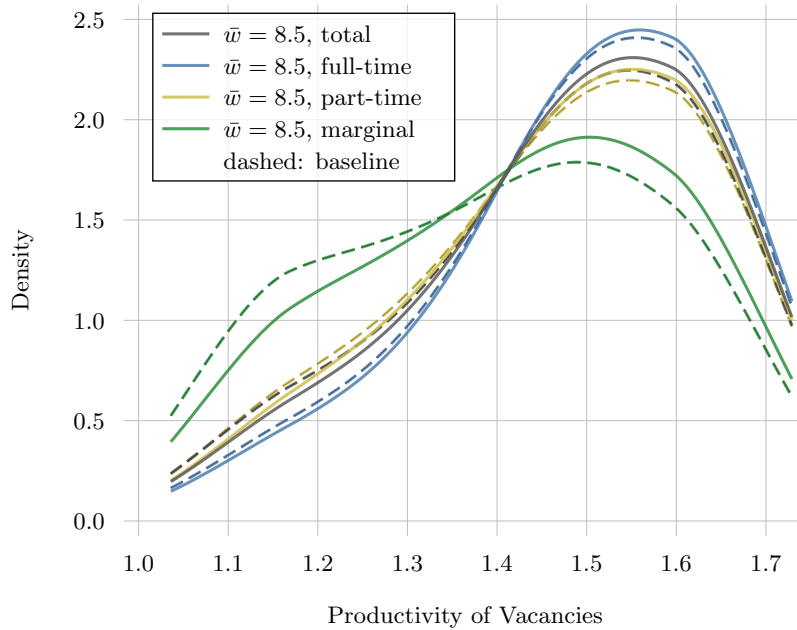
Note: This figure shows how the productivity distribution of vacancies offered by firms changes in response to the introduction of the minimum wage of 8.5 EUR. I exclude skill segments in which none of the minimum wages considered is binding for any job, i.e. where all wages in the baseline equilibrium are above 14.5 EUR.

Panel C shows how lifetime utility, income and earnings change relative to the baseline equilibrium. Although earnings increase substantially, income growth is much weaker due to the fact that many low-wage workers top up their earnings with government transfers and thus loose the majority of the earnings increase. Perhaps surprisingly, lifetime utility remains almost unchanged and is slightly negative for women. This is because the small increase in income (consumption) is counteracted by lower state utility as workers now work longer hours. Especially those workers who have a strong preference for or rely on marginal jobs with low working hours experience utility losses from the reallocation towards part- and full-time jobs.

To see this more clearly, panel D decomposes the average change in flow utility (closely correlated with lifetime utility) into the components of the utility function. While utility from consumption, $u(c)$, increases, hours-related utility, $\nu(s)$, decreases.[27]

4.4 Transitional Dynamics

In the presence of search frictions, the process of worker reallocation takes time. Workers whose jobs survive the introduction of the minimum wage will gradually transition to more productive firms or jobs with longer hours. More importantly, the minimum wage will make some jobs unprofitable. These workers become unemployed and finding a (better) job takes time. While worker reallocation pushes up output in the long-run, the short-run effects of introducing or raising the minimum wage may be significantly less desirable. It is thus paramount to study the transitional dynamics triggered by the minimum wage reform.[28]

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[27]Disutility from search plays almost no role.

[28]Appendix C documents how the transition path is computed.

FIGURE 10: Heterogeneous Effects by Sociodemographics

(A) Wage < 8.5 EUR (Baseline)

(B) Employment

(C) Earnings, Income & Lifetime Utility
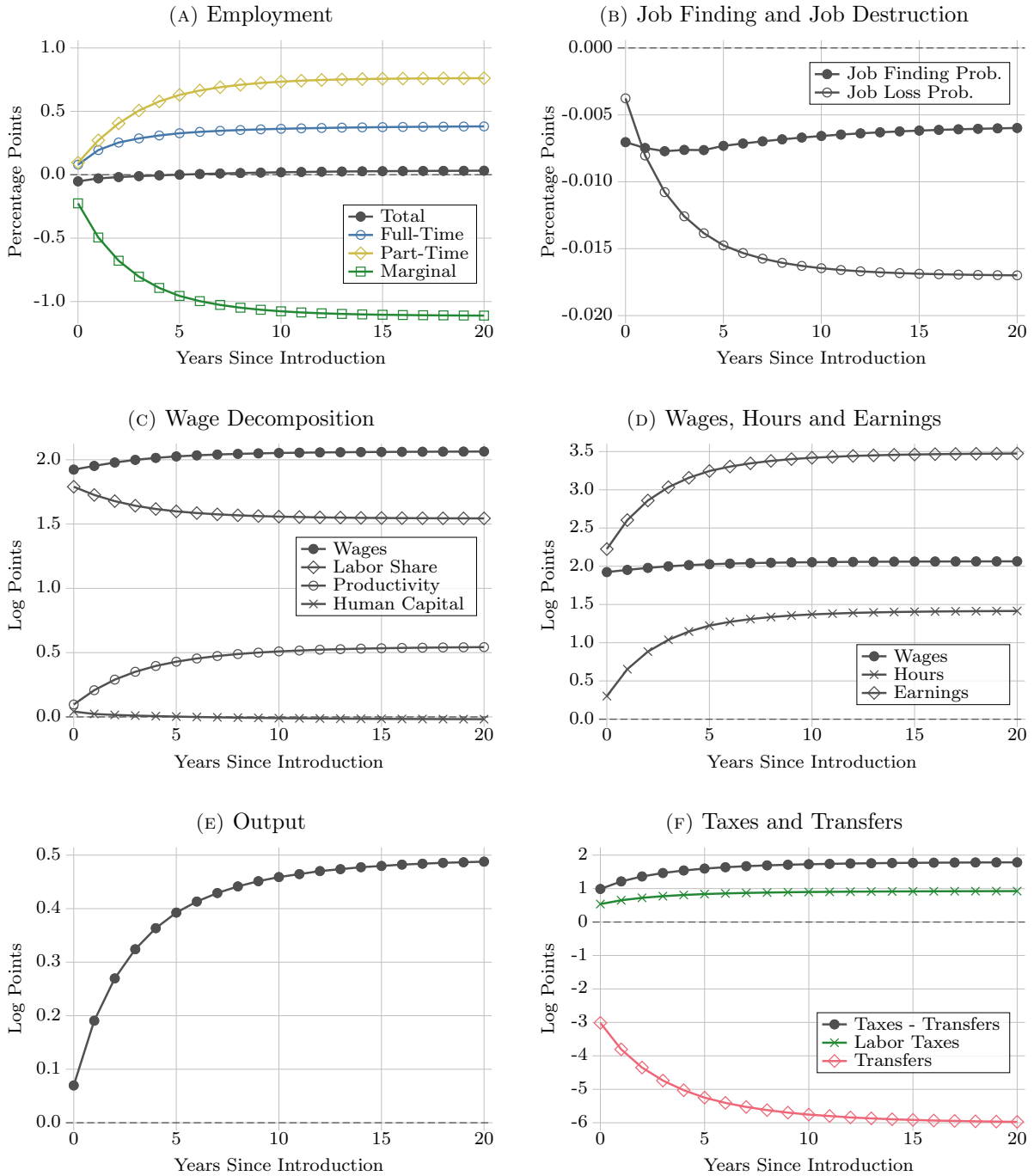
(D) Decomposition of Flow Utility
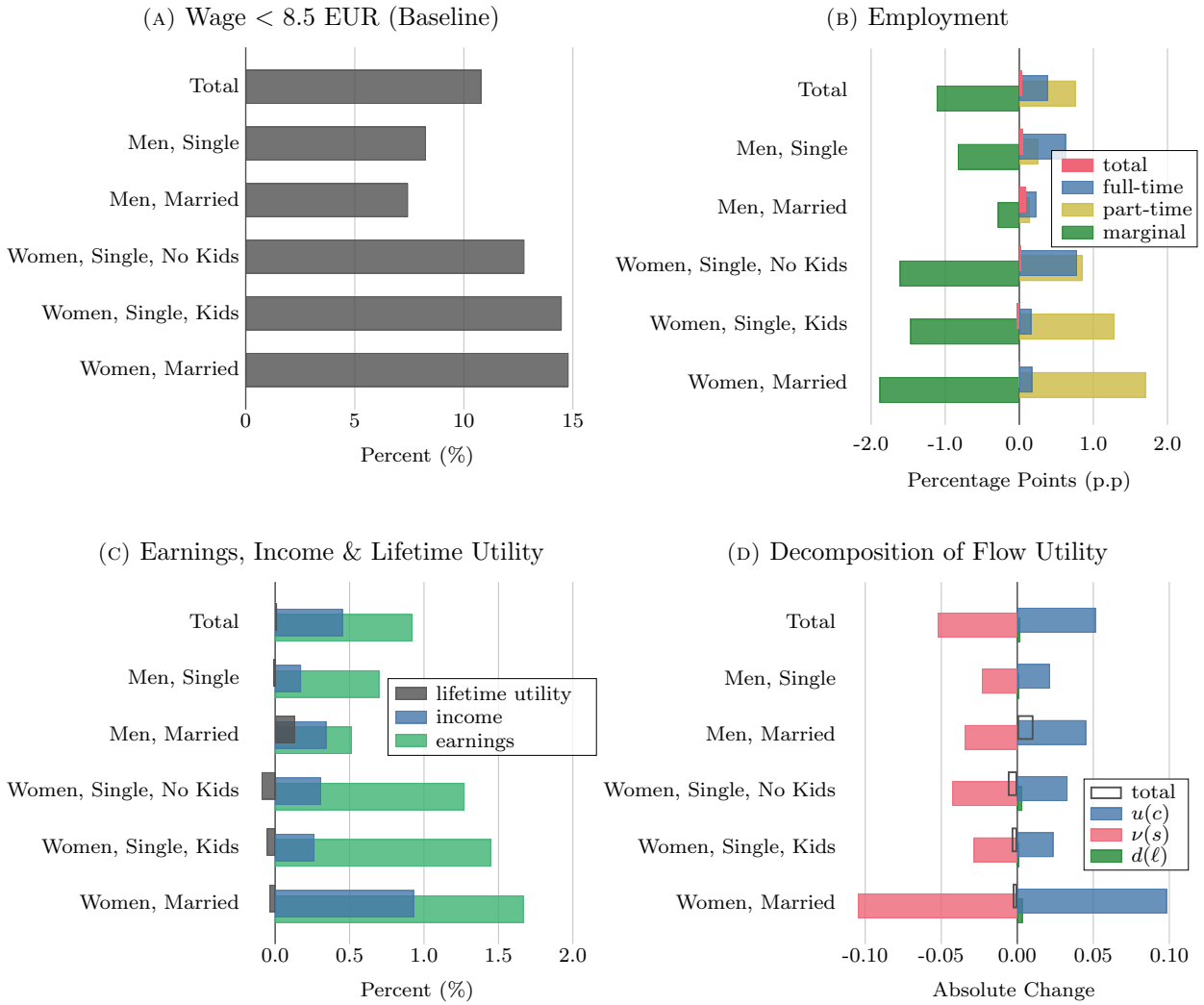
Note: This figure shows how the effects of the minimum wage of 8.5 EUR vary across sociodemographic groups. Panel A shows how many employed workers are affected by the minimum wage, panel B shows how the distribution of labor market states changes (the bars sum to zero). Panel C shows the relative change in average earnings, income/consumption and lifetime utility. Panel D decomposes the average change in flow utility into its components (see equation 20).

Figure 11 shows how the economy reacts to the reform. Panel A shows that there is indeed a drop in total employment as some jobs become unprofitable. It takes about five years until the employment response turns positive. The matgnitude of the initial layoff shock, however, is very small (0.052 percentage points). It takes roughly ten years for the shift towards part-time and full-time jobs unfold. This shift directly maps into a reduction of the average job destruction rate (panel B). The job finding rate out of unemployment exhibits rather weak transitional dynamics.

Panel C shows the evolution of average wages. Wages jump up immediately and increase only slightly over the following years. However, the decomposition of wage growth changes over time. Initially wage growth is almost entirely driven by lower profit margins and thus a higher average labor share. As time progresses, workers reallocate to more productive firms. Hence the profit margin recovers and workers' higher wages are increasingly the result of working for more

FIGURE 11: Dynamic Effects of the Initial Minimum Wage

(A) Employment

(B) Job Finding and Job Destruction

(C) Wage Decomposition

(D) Wages, Hours and Earnings

(E) Output

(F) Taxes and Transfers

Note: This figure shows the predicted changes in employment by job type (panel A), average job finding and job destruction probabilities (panel B), the components of wage growth (panel C), averages wages, hours and earnings (panel D), total output (panel E) and total taxes and transfers (panel F) following the introduction of a minimum wage of 8.5 EUR.

productive firms. We also see that wage growth is not driven by selection of relatively high-skill workers into employment.

Panel D shows that while both average wages and earnings increase immediately after the reform, earnings continue to grow substantially over the following ten to fifteen years. This is driven by the increase in average hours worked. As employment is essentially constant, the increase in average earnings translates into an increase in the government's revenue from labor income taxation. Similarly, low-skill workers receive lower transfers payments to top up their growing earnings (panel F).

As the initial dip in employment is negligible, total output increases monotonically following the reform (panel D). The minimum wage is low enough to prevent an initial dip due to the destruction of unprofitable matches. After five years, total output is already 0.39 log points above the pre-reform level. Taking into account that it takes a substantial amount of time until the new stationary equilibrium is reached, it is more informative to compare the net present value of output with and without the minimum wage. The minimum wage raises the net present value of output by 0.38 log points – about 77% of the log point difference in output between the two steady states.[29]

4.5 Comparison with Reduced-Form Evidence

I now briefly discuss how the model predictions line up with the available reduced-form evidence on the initial introduction of the minimum wage which can be seen as an independent test of the model. There are several studies documenting the short-run effects of the 2015 minimum wage reform using individual or regional variation in the bite of the minimum wage (e.g. Garloff, 2016; Caliendo et al., 2017; Holtemöller and Pohle, 2017; Burauel et al., 2020; Dustmann et al., 2020). The results of these studies boil down to the following points.

First, both hourly wages increased significantly and consistent with near full compliance from 2014 to 2016. Earnings grew by more than wages suggesting that work hours increased (Dustmann et al., 2020).[30]

Second, none of the afore-mentioned studies find significant adverse effect on overall employment. However, the minimum wage induced a shift from marginal jobs towards part-time and full-time jobs (Garloff, 2016; Holtemöller and Pohle, 2017). Caliendo et al. (2017) estimate that approximately 2.4% of marginal jobs were lost due to the minimum wage in the first year of the reform. vom Berge et al. (2016) document that the number of marginal workers dropped by about 2% and 4% from December 2014 to January and September 2015 respectively.[31] The model predicts that about 2.7% of marginal jobs were lost on impact and about 4.4% in the fall of 2015. In addition and consistent with the model, turnover rates decreased as both job finding and separation rates were reduced (Bossler and Gerner, 2016).

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[29]For this computation, I use the model discount factor to compute the net present values.

[30]Recall that I assume full compliance throughout the paper. There is some evidence of non-compliance in the first year after the introduction of the minimum wage (Burauel et al., 2020). However, the issue of non-compliance seems to have been rather transitory. In addition, earnings increased more strongly than wages over the first two years suggesting that, if anything, hours increased.

[31]Caliendo et al. (2017) include all marginal workers while vom Berge et al. (2016) only include those workers where the marginal job is the main job. Note that I use the same definition as vom Berge et al. (2016).

Third, there is also evidence that the minimum wage reallocated workers to larger, more productive firms (Dustmann et al., 2020) . Quantitatively, reallocation to more productive firms seems to have happened slightly quicker in the data than in the model. (Dustmann et al., 2020) attribute about one quarter of the increase in wages from 2014 to 2016 to reallocation to better firms. In the model, about 15% of the wage gain comes from reallocation. This is likely due to the fact that job destruction in the model does not depend on human capital or firm productivity. In the data, the probability of job destruction decreases in both. The broad pattern, however, is consistent with this empirical finding. After five years and in the new steady state, 25% of the wage gain is driven by productivity gains.

In sum, the estimated model captures all these effects qualitatively and does a good job of replicating them quantitatively. The fact that the model not only matches well the labor market moments in the pre-reform period, but is also broadly consistent with the rich reduced-form evidence on the minimum wage reform lends credibility to the optimal policy analysis in section 5.

5 Counterfactuals: Increasing the Minimum Wage

In this section, I use the structural model to analyze how increasing the minimum will impact employment and output and welfare. First, I will analyze the long-run employment and output effects (section 5.1). Second, I will analyze the entire transition path and compare short- and long-run effects (section 5.2). Third, I will discuss heterogeneity in welfare effects of minimum wages (section 5.3).
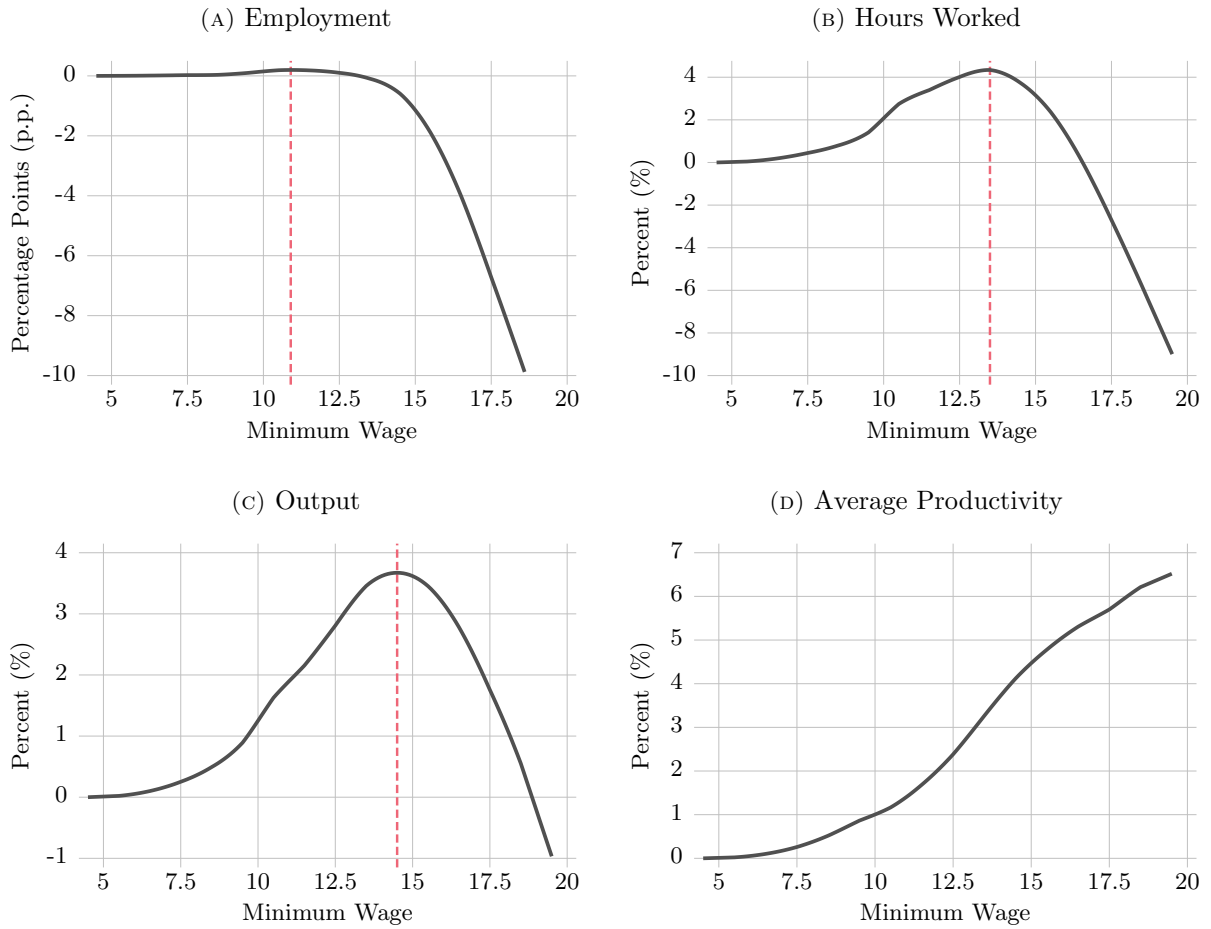
5.1 Long-Run Effects

I first take a long-run perspective by comparing the stationary equilibrium that emerges for different minimum wages to the baseline equilibrium. Figure 12 shows steady-state employment, hours worked, output and average productivity as a function of the minimum wage. Panel A shows that total employment, i.e. the share of employed workers, is a non-monotonic function of the minimum wage. Employment is maximized at a minimum wage of 11.0 EUR (Kaitz = 60%). Employment does not drop below the baseline level for minimum wages below 12.8 EUR (Kaitz = 70%). Quantitatively, the positive employment effect of moderate minimum wages is very small (≤ 0.2 p.p.) while the decline in employment for high minimum wages is quite steep.

Panel B shows that the minimum wage not only affects the extensive margin of employment but also the average employment level. While the number of employed workers increases only slightly for moderate minimum wage hikes, total hours worked do increase significantly. Importantly, the hours-maximizing minimum wage of 13.5 EUR (Kaitz = 73%) is considerably higher than the employment-maximizing minimum wage. Hence, even though employment starts to decline, total hours continue to increase because the average employment level increases. At the hours-optimum, total hours worked are 4.3% above the baseline level.

This increase in total hours worked implies that the output-maximizing wage is significantly higher than the employment-maximizing one and that the minimum wage can lead to long-run output growth. Indeed, Panel C shows that output increases considerably in the minimum wage

FIGURE 12: Long-Run Minimum Wage Effects

(A) Employment

(B) Hours Worked

(C) Output

(D) Average Productivity

Note: This figure shows the predicted long-run minimum wage effects on employment, hours worked, output and average productivity in panels A through D respectively. The red dashed lines in panels A, B and C indicate the maximum.

up until 14.5 EUR (Kaitz = 79%). At the optimum, total output is 3.7% higher compared to the baseline without a minimum wage. Beyond that point, output starts to decline as total employment drops sharply. Panel D shows that *average* firm productivity increases monotonically in the minimum wage. This second margin of reallocation explains why the output-optimum is above the hours-optimum. At the output-maximizing minimum wage, jobs are on average 4.1% more productive.

In order to understand what drives these minimum wage effects in Figure 12, I fix (a) workers' search policies, (b) the expected surplus of meeting a firm, (c) the productivity-hours distribution of vacancies, and (d) the mass of vacancies at the corresponding baseline levels. Figure 13 illustrates the results in each of these partial equilibrium scenarios.

Panel A shows that endogenous search effort and, in particular, the increase in the expected surplus of meeting a firm is responsible for the lack of disemployment effect for moderate minimum wages in general equilibrium. If workers' are not allowed to adjust their search behavior, employment decreases monotonically as the minimum wage exceeds 7.5 EUR. If workers are allowed to re-optimize, but the surplus of contacting a firm is held at its baseline level, the drop in employment is even more pronounced. This is because workers now reduce their search effort

FIGURE 13: Long-Run Minimum Wage Effects – Mechanisms

(A) Employment

(B) Hours Worked

(C) Output

(D) Average Productivity
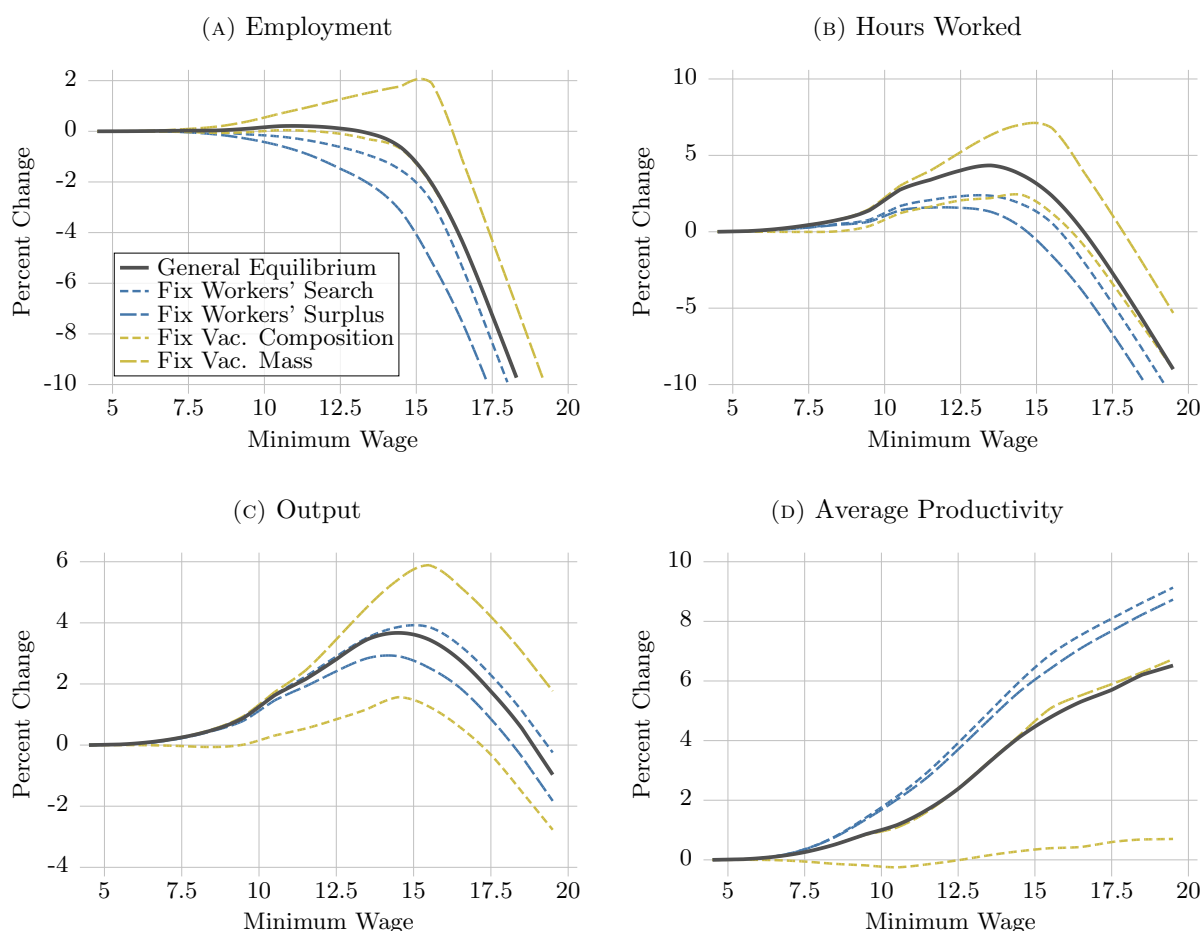
Note: This figure shows the predicted relative changes in total employment, total hours worked, total output and average productivity as a function of the minimum wage for different scenarios. The blue short-dashed line shows a scenario where workers' search effort is held fixed at the pre-reform levels. The blue long-dashed line refers to a scenario that allows search effort to adjust, but keeps the surplus of meeting a firm constant. The yellow short-dashed line displays a scenario where the hours-and productivity distribution of vacancies is held fixed while the total mass of vacancies is allowed to adjust. The yellow long-dashed line refers to a scenario where the distribution of vacancies is flexible but the mass of vacancies is fixed.

as firms post fewer vacancies and the aggregate contact rate drops (search effectiveness). In contrast, when the number of vacancies is held constant and workers' can adjust their search effort, total employment increases significantly in the minimum wage. The negligible employment effects for moderate minimum wages is therefore not due to a muted reduction of firms' vacancy posting, but rather the net effect of two off-setting forces. In other words, changes in the demand and supply of labor largely offset each other for moderate minimum wage hikes. As the minimum wage approaches 15 EUR, however, broad non-employability of low-skill workers kicks in and employment falls.[32]

Changes in the productivity and employment level distribution of vacancies have only a small impact on total employment, they do impact the response of both total hours worked and average productivity (Panels B and D). It is clear that the profitability of low-productivity jobs declines relative to that of high-productivity jobs for a given minimum wage. Optimal behavior

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[32]Note that in the scenario where the mass of vacancies is held fixed, I exclude non-employable vacancies.

FIGURE 14: Productivity Distribution of Vacancies
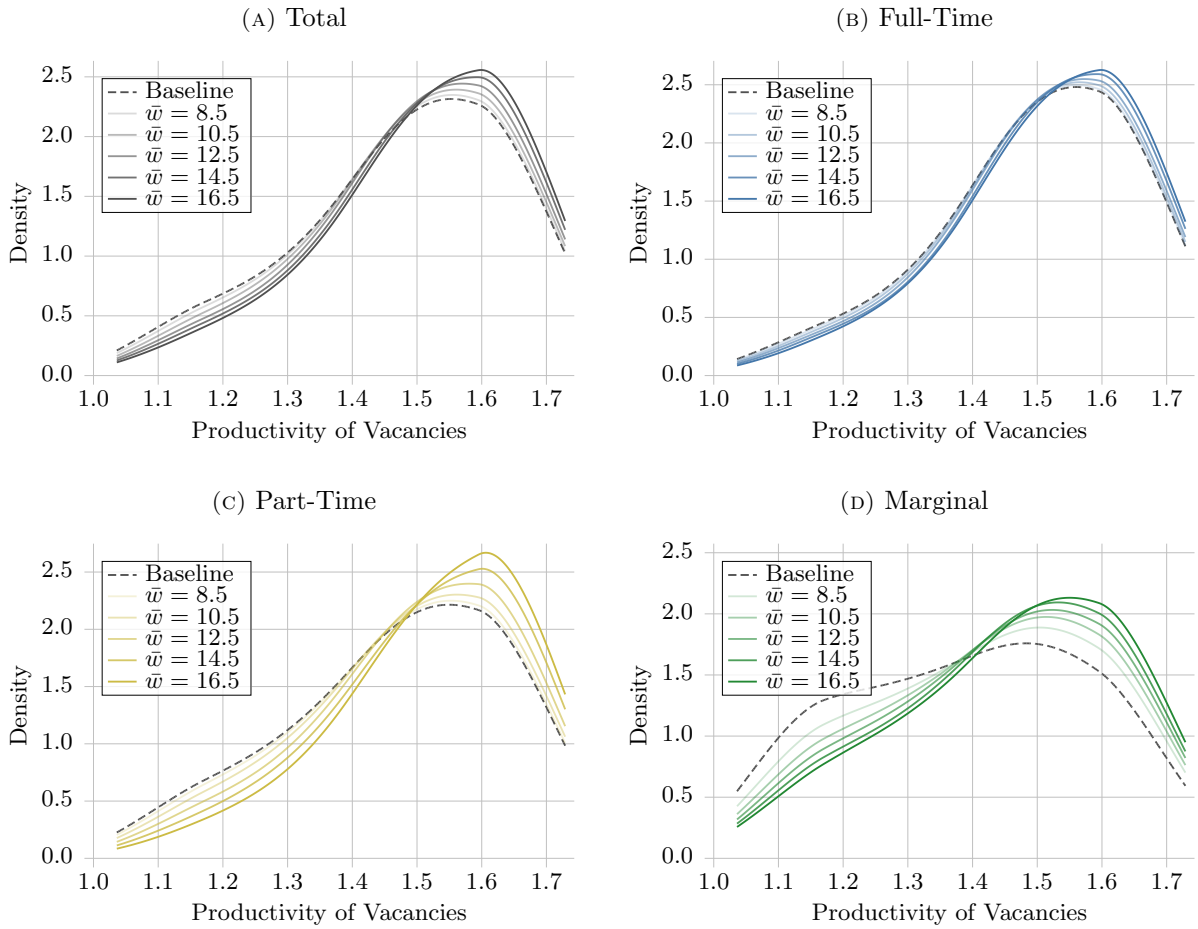
(A) Total

(B) Full-Time

(C) Part-Time

(D) Marginal

Note: This figure shows how the productivity distribution of vacancies offered by firms changes with the minimum wage. Panel A shows the distribution for all vacancies. Panels B, C and D show the productivity distribution of full-time, part-time and mini-job vacancies respectively. I exclude skill segments in which none of the minimum wages considered is binding for any job, i.e. where all wages in the baseline equilibrium are above 16.5 EUR.

by firms implies that asymmetric declines in profitability lead to asymmetric vacancy reductions. This can be seen in Panel A of Figure 14 which plots the equilibrium productivity distribution of all vacancies for selected minimum wages. Clearly, the minimum wage shifts this distribution to the right. Conditional on meeting a firm, average firm productivity thus increases and workers move to more productive firms. Panel D shows that fixing the composition of vacancies kills the positive productivity effect of minimum wages.

Panels B, C and D of Figure 14 further show that full-time vacancies are relatively more productive than vacancies for part-time and especially marginal jobs. Hence, the response of firms' vacancy posting is not only asymmetric in terms of productivity but also employment levels. Therefore, fixing the composition of vacancies also mutes the effect on total hours worked as there is less reallocation towards high-hours jobs.

While firms' vacancy posting decisions drive up total hours worked and average productivity in response to a minimum wage hike, the optimal response of workers increases hours worked but decreases average firm productivity. On the one hand, a binding minimum wage increases the relative value of high-hours jobs and thus increases the incentives of marginal and part-time

workers to engage in on-the-job search for a job with a higher employment level. This is because, for a given increase in the hourly wage, earnings and therefore consumption growth is higher for jobs with higher employment levels.[33] Therefore, fixing fixing workers' search effort or their surplus of meeting a firm at the baseline levels reduces the positive hours effect of minimum wages by about 50%. Conversely, a binding minimum wage reduces the surplus of working for a high productivity firm as the minimum wage eliminates or reduces productivity-related wage differentials. This reduces the incentives for on-the-job and the probability that a worker at a low-productivity firm will accept a job offer from a high-productivity firm (with the same employment level). Hence, fixing this adjustment mechanism amplifies the positive productivity effect of increasing the minimum wage.

Finally, Panel C shows how total output evolves in these partial equilibrium scenarios. Allowing firms' to adjust their vacancy posting decisions reduces output because the total mass of vacancies and thus employment drops but increases output because the hours- and productivity distribution of vacancies shifts toward more productive and full-time jobs. Allowing workers' to adjust their search effort has only small effects on total output as it has a positive effect on employment and hours worked but a negative effect on average productivity.

In sum, the steady state analysis shows that increasing the minimum creates a trade-off between employment and output. Policy makers can use the minimum wage to improve the average productivity and employment level of jobs and thereby average output per job. However, the model predicts that, for minimum wages beyond a Kaitz index of 60%, improved job composition has to be traded off against total employment.

## 5.2 Transition Dynamics: Long-Run Gain vs. Short-Run Pain

The steady state comparisons shows that reallocation is crucial in order to understand the effects increasing the minimum wage. In a world of search frictions, reallocation will take time and can be quite painful. Figure 15 shows how many jobs in the baseline equilibrium will become unprofitable for different minimum wages. The higher the minimum wage, the more jobs will be destroyed following the minimum wage hike. While initial job destruction is not important for minimum wages below 10 EUR, it is increasingly important for higher minimum wages. At the long-run output maximum of 14.5 EUR, for example, over 10% of all jobs are destroyed initially.

These workers become unemployed and have to find a new (better) job which takes time and effort. Taking search frictions seriously thus requires one to analyze the entire transition path following a minimum wage hike.[34] Figure 16 shows how the unemployment rate and output evolve following minimum wage hikes of different magnitudes. In particular, the black line corresponds to the observed minimum wage reform (8.5 EUR) and the green, yellow and red lines correspond to the long-run employment maximum (11 EUR), the highest minimum wage without long-run disemployment effects (12.8 EUR), and the long-run output-maximizing minimum wage (14.5 EUR) respectively.

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[33]Note that the minimum wage does not change the disutility of working long hours. Hence, whatever the initial relative value of full-time jobs, it will increase in the minimum wage. This is consistent with the theoretical and empirical results presented by Doppelt (2019) who analyzes a stylized model where workers can choose the number of hours worked.

[34]I allow for a notice period of one quarter. See Appendix C for details on how to compute the transition path.

FIGURE 15: Initial Job Destruction
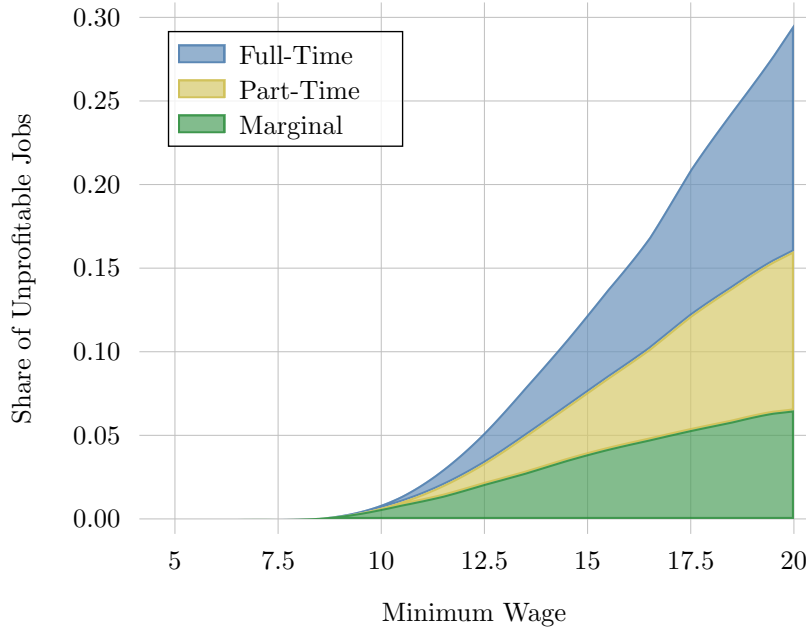
Note: This figure shows what share of jobs in the baseline equilibrium becomes unprofitable for different minimum wages. The different areas decompose the total share into different employment levels.

As expected, we see significant spikes in the unemployment rate at the time the minimum wage is imposed (Panel A). At the long-run employment-maximizing minimum wage of 11 EUR, the unemployment rate increases by about 1.5 percentage points on impact (increase of 20%). While a minimum wage of 12.8 EUR does not lead to disemployment effects in the long-run, it does so in the short- and medium run as the economy takes about 10 to 15 years to convergence to the new stationary equilibrium. At the long-run output-maximizing minimum wage of 14.5 EUR, the unemployment rate more than doubles following the reform and takes three years to fall below 10%. After five years, the unemployment rate is still 24% above the baseline and 15% above the new long-run unemployment rate.

Panel B shows that output gains also take time to materialize. For minimum wages above 11 EUR, the minimum wage hike forces the economy into a recession. On the to the long-run output maximum, output falls below its baseline level for almost two years. Nevertheless, output gains take less time to kick in than it takes the unemployment rate to drop. This is because high short-run unemployment is mostly driven by workers at the bottom end of the skill distribution and the contribution of these workers to total output is relatively small. The trade-off between output and employment thus becomes magnified in the short- and medium run.

In order to formalize how policy makers' planning horizon affects their assessment of higher minimum wages, Figure 17 shows how the average discounted unemployment rate (panel A) and the net present value of output (panel B) evolve as a function of the minimum wage and for different time horizons T. Panel A shows the average discounted unemployment rate between

FIGURE 16: Minimum Wage Effects Along the Transition Path

(A) Unemployment
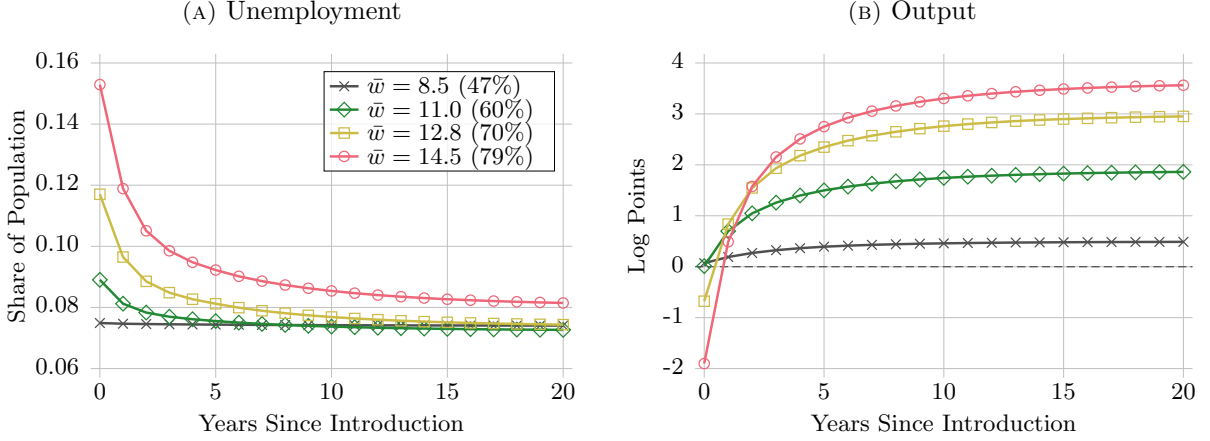
(B) Output

Note: This figure shows how the unemployment rate and output evolve following minimum wage hikes of different magnitude – always starting at the baseline equilibrium without a minimum wage. I assume that the minimum wage hike is announced one quarter before it becomes binding.

$t = 0$ and $t = T$:

$$\frac{1}{\sum_{t=0}^{T} \beta^t} \sum_{t=0}^{T} \beta^t u_t(\bar{w}) \tag{24}$$

where $u_t(\bar{w})$ is the unemployment rate t periods after the minimum wage is raised from zero to $\bar{w}$ and the workers' discount factor ($\beta = 0.98$) is used. Panel B shows the log difference in the net present value of output relative to the baseline without a minimum wage for time horizon T:

$$\log\left(\sum_{t=0}^{T} \beta^t Y_t(\bar{w})\right) - \log\left(\sum_{t=0}^{T} \beta^t Y_t(0)\right) \tag{25}$$

where $Y_t(\bar{w})$ is total output in period t after a minimum wage of $\bar{w}$ was introduced. The lighter the line, the longer the time horizon T. The darkest line corresponds to $t = 0$ and the red dashed line corresponds to $T = \infty$. The lines in between show the change in output and the unemployment rate 1, 2, 5, 10 and 20 years after the introduction of the minimum wage.

Panel A of Figure 17 shows that there is no binding minimum wage that increases the average discounted unemployment rate – even for an infinite time horizon. The long-run reduction in the unemployment rate is not big enough to outweigh higher short-run unemployment rates. The long-run net present value of output is maximized at a minimum wage of 14.2 EUR and 2.68 log points above the net present value of output without a minimum wage. Adopting a five and two year horizon, the output maximizing minimum wage drops to 13.5 EUR and 12.2 EUR respectively with smaller but still significant discounted output gains of 1.43 and 0.67 log points.

5.3 Who Benefits from High Minimum Wages?

As a final step, I use the model to analyze how increasing the minimum wage affects lifetime utility of workers. Panel A of Figure 18 shows that average lifetime utility in the population

FIGURE 17: Discounted Output and Employment Effects

(A) Unemployment
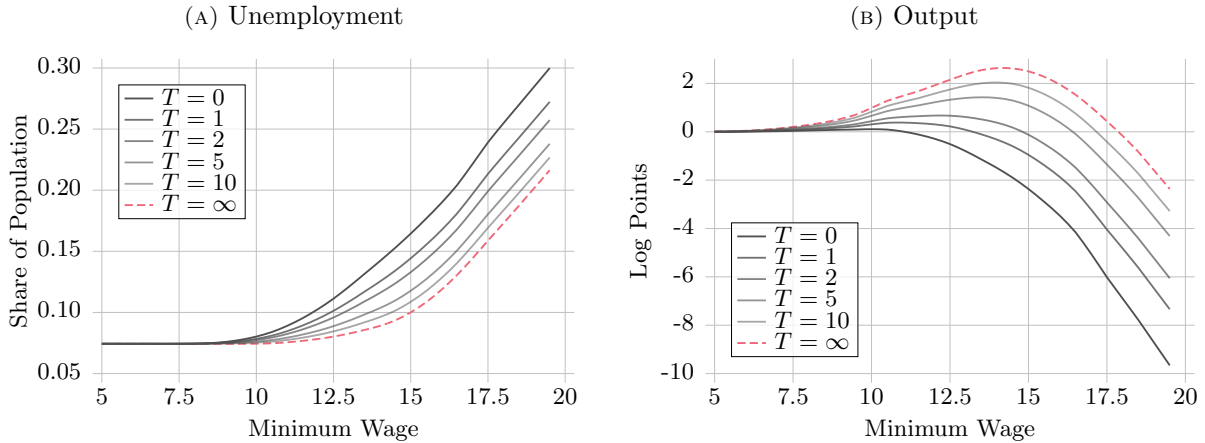
(B) Output

Note: This figure shows and the average discounted unemployment rate (panel A) and the net present value of output (panel B) of different minimum wage policies for different time horizons. Lighter lines correspond to longer time horizons. The red dashed line corresponds to an infinite time horizon.

increases up until a minimum wage of over 16 EUR. However, at the per capita optimum, a significant share of low-skill workers experiences large welfare losses. For the bottom 5, 10 or 20 percent of the human capital distribution, average lifetime utility peaks between 13 and 14 EUR and declines sharply to the right of the optimum. This is because high minimum wages make low-skill workers unemployable and forces them into long-term unemployment. While low-skill workers are the ones who benefit the most from increasing the minimum wage, they also suffer the most if the minimum wage is set so high that they become unemployable.

Panel B shows how the lifetime utility changes for different sociodemographic groups. As with the initial minimum wage, welfare gains are not distributed equally. While average lifetime utility of men and single women without kids grows strongly with higher minimum wage levels, single women with kids and married women do not benefit from the reallocation effects in terms of their lifetime utility. The latter actually experience small welfare losses for moderately high minimum wages between 10 and 14 EUR. Reallocation away from low-hours jobs comes at a disutility cost of longer working hours. This disutility is estimated to be substantially larger for single women with kids and married women reflecting the large share of non-full-time jobs among these workers in the baseline equilibrium. Time constraints due to childcare obligations that feature into this disutility thus interact with the minimum wage.

In the end, the minimum wage remains a crude policy tool that is not targeted at certain skill levels or sociodemographics. Hence, different sub-populations do not benefit equally from higher minimum wages even though aggregate welfare – measured by the average lifetime utility across all workers – increases in the minimum wage.

5.4 Different Baseline: Increasing the Minimum Wage

Thus far, the analysis was conducted using the economy without a minimum wage as the baseline. While this is useful to understand the short- and long-run effects, most countries – including Germany in 2020 – already have some positive minimum wage in place. I now assume that the

FIGURE 18: Heterogeneous Welfare Effects

(A) By Skill Group

(B) By Sociodemographics
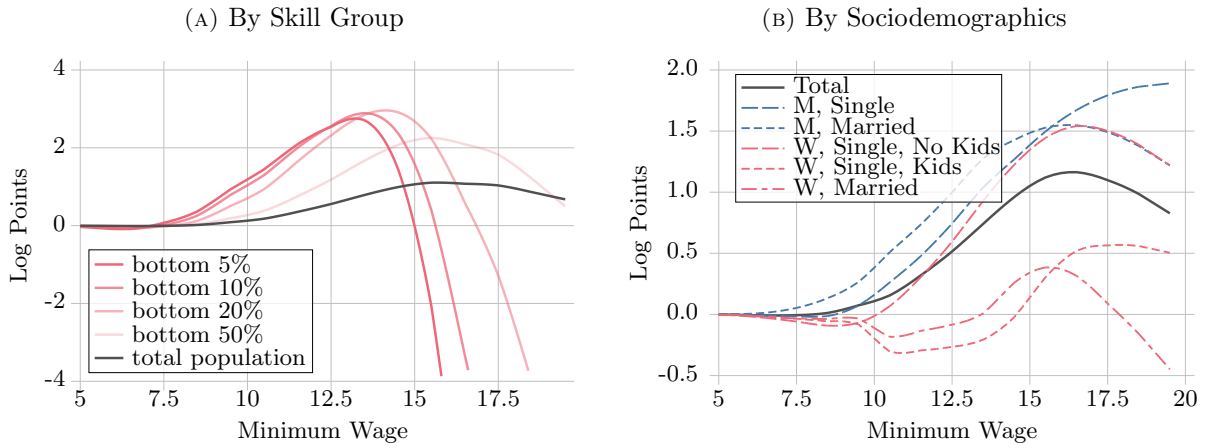
Note: This figure shows how the minimum wage changes average lifetime utility of different sub-populations. Panel A distinguishes between different parts of the human capital distribution and Panel B presents the effects by sociodemographic characteristics.

economy has already converged to the stationary equilibrium with a minimum wage of 8.5 EUR – the initial level set in 2015. Figure **??** suggests that this is a reasonable assumption as both output and employment are close to the new stationary equilibrium five years after the reform. Analogous to Figure **??**, Figure 19 shows how the net present value of output and the average unemployment rate evolve at different time horizons as a function of the minimum wage using the 8.5 EUR scenario as the point of departure. However, the results change only marginally. The reason for this is that a minimum wage of 8.5 EUR is still relatively low and does not alter the starting point enough to affect the transition quantitatively. Hence, policy makers still face the same trade-offs when thinking about raising the minimum wage with a positive minimum wage already in place.

FIGURE 19: Optimal Minimum Wages with Baseline at 8.5 EUR

(A) Output Change

(B) Unemployment Rate
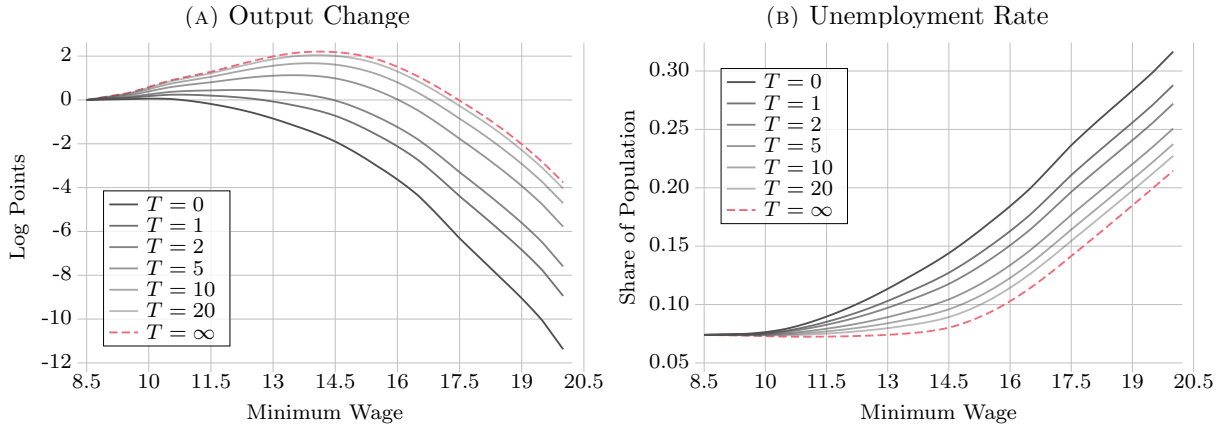
Note: This figure shows the predicted changes in output (panel A) and employment (panel B) of different minimum wage policies for different time horizons after the reform. The pre-reform minimum wage is 8.5 EUR. Lighter lines correspond to later points in time.

6 Discussion & Conclusion

The main goal of this paper is to construct a rich quantitative search-matching model that is consistent with recent reduced-form evidence on employment and reallocation effects of observed minimum wages (Cengiz et al., 2019; Dustmann et al., 2020) and can be used to analyze how output, employment and welfare react to increasing the minimum wage. This analysis is motivated by recent proposals to increase minimum wages in developed countries and the lack of quantitative structural models that can inform policy makers. While more research is clearly necessary to better understand how the economy would react to higher minimum wages, this paper takes multiple steps in that direction.

I show that a rich model with two-sided heterogeneity, endogenous search and vacancy posting, a realistic tax- and transfer system and multiple employment levels can not only match important aspects of pre-reform micro data but also replicate the available reduced form evidence on the German minimum wage (Dustmann et al., 2020). In particular, the minimum wage of 8.5 EUR had negligible employment effects while increasing productivity, wages and the average employment level.

I use the estimated and tested model to analyze how increasing the minimum wage affects employment and output in the short- and long-run. The analysis offers at least four important insights. First, total employment increases slightly in the minimum wages up to a Kaitz index of 60% as increasing search effort by workers offsets decreasing vacancy posting by firms. Allowing not only firms but also workers to changes in the respective value of an employment relationship can thus explain why past and current minimum wage hikes have not lead to significant disemployment effects Cengiz et al. (2019). Second, while increasing the minimum wage above 60% of the median wage leads to a decline in the number of jobs, total hours worked continue to increase and peak at a Kaitz index of 73%. This shows that taking into account differences in employment levels and the fact that relatively many low-wage jobs are also low-hours jobs is crucial in order to understand how the minimum wage affects the composition of jobs and hence

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average output per job. Before the introduction of the minimum wage in Germany, full-time jobs accounted for only one third of all jobs affected by the minimum wage of 8.5 EUR (Kaitz index of 47%). Moreover, the composition of jobs shifts towards more productive firms as low-productivity firms become relatively less profitable. While most of the literature on minimum wages has focused on (dis-)employment effects of minimum wages, these results suggest that analyzing job composition and output effects is equally important. In the model, a Kaitz index of 79% maximizes steady state output in the economy. Third, I show that it is paramount to take search frictions seriously and analyze the transition dynamics following a minimum wage hike. Since large minimum wage hikes destroy jobs on impact, unemployment rates spike. Search frictions and lower vacancy posting make finding a (better) job very time-consuming. Long run output gains thus go hand-in-hand with substantial short- and medium-run unemployment rates and even recessions of up to two years. Fourth, not all workers benefit from higher minimum wages. Low-skill workers become non-employable and are stuck in long-term unemployment. Moreover, many women who rely on marginal jobs that are replaced by part-time and full-time jobs experience substantially smaller welfare gains compared to men. The results thus suggest that providing adequate child care opportunities is important to allow all workers to benefit from higher minimum wages.

By showing that increasing minimum wages above observed levels can lead to sizable long-run output effects with muted disemployment effects, this paper opens up new avenues for future research. First, my paper – as well as the literature as a whole – abstracts from real-world features that may become more important for large minimum wages. First, I suspect that, at some point, high minimum wages will lead firms in the tradeable sector to consider moving their workforce to countries with lower wage floors. Empirical analyses of past reforms point to somewhat stronger disemployment effects in the tradeable sector (e.g. Cengiz et al., 2019). As wages in the tradeable sector are relatively high even without minimum wages, this distinction has been quantitatively unimportant. Analyzing to what extent and at what point firms decide to relocate to other countries will be important to assess the costs and benefits of higher minimum wages. As for taxation, international cooperation may become important for minimum wage laws.

Second, more work is needed to assess whether the large output effects are robust to changes in the production function. Complementarities between low- and high-skill tasks can reduce demand for high-skill jobs as low-skill jobs become non-profitable, but may also limit the decline in vacancy posting for low-skill jobs as they are required for more important high-skill tasks. In addition, it will be fruitful to investigate how firms' investment decisions are affected by the minimum wage. Will high minimum wages lead firms to replace labor with capital or lead to higher productivity growth.

Third, exploring the effects on endogenous human capital accumulation seems important. On the one hand, higher minimum wages may decrease workers' incentives to invest in their education as wage differentials are reduced. On the other hand, the disappearance of jobs in low-skill segments of the labor market will increase the human capital accumulation.

Fourth, the inter-temporal trade-offs associated with higher minimum wages deserve more attention. At what speed should policy makers implement increases in the minimum wage

in order to mitigate the short-run losses while still benefiting from productivity- and output-enhancing reallocation effects?

Finally, the effects of the minimum wage interact with other labor market policies such as the design of unemployment insurance or earned income tax credits. As both lower unemployment benefits and higher minimum wages affect workers' surplus of employment, the optimal generosity of the social safety net and the level of the minimum wage should be determined jointly.

References

ABOWD, J., F. KRAMARZ, AND D. MARGOLIS (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.

ACEMOGLU, D. (2001): "Good Jobs versus Bad Jobs," *Journal of Labor Economics*, 19, 1–21.

ARNOUD, A., F. GUVENEN, AND T. KLEINEBERG (2019): "Benchmarking Global Optimizers," Working Paper 26340, National Bureau of Economic Research.

BAGGER, J., F. FONTAINE, F. POSTEL-VINAY, AND J.-M. ROBIN (2014): "Tenure, Experience, Human Capital, and Wages: A Tractable Equilibrium Search Model of Wage Dynamics," *American Economic Review*, 104, 1551–96.

BAGGER, J. AND R. LENTZ (2018): "An Empirical Model of Wage Dispersion with Sorting," *The Review of Economic Studies*, 86, 153–190.

BLÖMER, M., N. GUERTZGEN, L. POHLAN, H. STICHNOTH, AND G. J. VAN DEN BERG (2020): "Unemployment Effects of the German Minimum Wage in an Equilibrium Job Search Model," .

BONHOMME, S., T. LAMADON, AND E. MANRESA (2019): "A Distributional Framework for Matched Employer Employee Data," *Econometrica*, 87, 699–739.

BONTEMPS, C., J.-M. ROBIN, AND G. J. VAN DEN BERG (1999): "An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms," *International Economic Review*, 40, 1039–1074.

BOSSLER, M. AND H.-D. GERNER (2016): "Employment Effects of the New German Minimum Wage: Evidence from Establishment-level Micro Data," IAB-Discussion Paper 10/2016, Institute for Employment Research.

BRENZEL, H., J. CZEPEK, H. KIESL, B. KRIECHEL, A. KUBIS, A. MOCZALL, M. REBIEN, C. ROETTGER, J. SZAMEITAT, A. WARNING, ET AL. (2016): "Revision of the IAB Job Vacancy Survey: Backgrounds, Methods and Results," *IAB-Forschungsbericht*.

BURAUEL, P., M. CALIENDO, M. M. GRABKA, C. OBST, M. PREUSS, C. SCHRÖDER, AND C. SHUPE (2020): "The Impact of the German Minimum Wage on Individual Wages and Monthly Earnings," *Jahrbücher für Nationalökonomie und Statistik*, 240, 201–231.

BURDETT, K. AND D. T. MORTENSEN (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 257–273.

CAHUC, P., F. POSTEL-VINAY, AND J.-M. ROBIN (2006): "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 74, 323–364.

CALIENDO, M., A. FEDORETS, M. PREUSS, C. SCHRÖDER, AND L. WITTBRODT (2017): "The Short-Run Employment Effects of the German Minimum Wage Reform," IZA Discussion Papers 11190, Institute for the Study of Labor (IZA).

CARD, D., J. HEINING, AND P. KLINE (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality," *The Quarterly Journal of Economics*, 128, 967.

CENGIZ, D., A. DUBE, A. LINDNER, AND B. ZIPPERER (2019): "The Effect of Minimum Wages on Low-wage Jobs," *The Quarterly Journal of Economics*, 134, 1405–1454.

CHETTY, R. (2008): "Moral Hazard versus Liquidity and Optimal Unemployment Insurance," *Journal of Political Economy*, 116, 173–234.

DI ADDARIO, S., P. KLINE, R. SAGGIO, AND M. SOLVSTEN (2020): "It Ain't Where You're From, It's Where You're at: Hiring Origins, Firm Heterogeneity, and Wages," Tech. Rep. 104-20, IRLE Working Paper.

DOPPELT, R. (2019): "Minimum Wages and Hours of Work," *mimeo.*

DUBE, A. (2019): ""Impacts of minimum wages: Review of the international evidence," *Independent report, https://www. gov. uk/government/publications/impacts-of-minimum-wages-review-of-the-international-evidence.*

DUBE, A., T. W. LESTER, AND M. REICH (2016): "Minimum Wage Shocks, Employment Flows, and Labor Market Frictions," *Journal of Labor Economics*, 34, 663–704.

DUSTMANN, C., A. LINDNER, U. SCHÖNBERG, M. UMKEHRER, AND P. VOM BERGE (2020): "Reallocation Effects of the Minimum Wage," CReAM Discussion Paper Series 2007, Centre for Research and Analysis of Migration (CReAM), Department of Economics, University College London.

ECKSTEIN, Z. AND K. I. WOLPIN (1990): "Estimating a Market Equilibrium Search Model from Panel Data on Individuals," *Econometrica*, 783–808.

ENGBOM, N. AND C. MOSER (2018): "Earnings Inequality and the Minimum Wage: Evidence from Brazil," *Federal Reserve Bank of Minneapolis-Opportunity and Inclusive Growth Institute Working Paper*, 7, 18–50.

FLINN, C. J. (2006): "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates," *Econometrica*, 74, 1013–1062.

GARLOFF, A. (2016): "Side Effects of the New German Minimum Wage on (Un-)Employment : First Evidence from Regional Data," IAB Discussion Paper 201631, Institut für Arbeitsmarkt- und Berufsforschung (IAB), Nürnberg.

HAGEDORN, M., F. KARAHAN, I. MANOVSKII, AND K. MITMAN (2019): "Unemployment Benefits and Unemployment in the Great Recession: The Role of Macro Effects," Working Paper 19499, National Bureau of Economic Research.

HARASZTOSI, P. AND A. LINDNER (2019): "Who Pays for the Minimum Wage?" *American Economic Review*, 109, 2693–2727.

HARTUNG, B., P. JUNG, AND M. KUHN (2018): "What Hides behind the German Labor Market Miracle? Unemployment Insurance Reforms and Labor Market Dynamics," CEPR Discussion Paper DP13328, Centre for Economic Policy Research.

HOLTEMÖLLER, O. AND F. POHLE (2017): "Employment Effects of Introducing a Minimum Wage: The Case of Germany," Tech. rep., IWH Discussion Papers.

JOLIVET, G., F. POSTEL-VINAY, AND J.-M. ROBIN (2006): "The Empirical Content of the Job Search Model: Labor Mobility and Wage Distributions in Europe and the US," *European Economic Review*, 50, 877 – 907.

JÄGER, S., B. SCHOEFER, S. YOUNG, AND J. ZWEIMÜLLER (2020): "Wages and the Value of Nonemployment," *The Quarterly Journal of Economics*, qjaa016.

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KRAUSE, M. U. AND H. UHLIG (2012): "Transitions in the German Labor Market: Structure and Crisis," *Journal of Monetary Economics*, 59, 64 – 79, carnegie-NYU-Rochester Conference Series on Public Policy at New York University on April 15-16, 2011.

KREBS, T. AND M. SCHEFFEL (2013): "Macroeconomic Evaluation of Labor Market Reform in Germany," *IMF Economic Review*, 61, 664–701.

——— (2016): "Structural Reform in Germany," Tech. rep., IZA Discussion Papers 9787.

LJUNGQVIST, L. AND T. J. SARGENT (1998): "The European Unemployment Dilemma," *Journal of Political Economy*, 106, 514–550.

MARINESCU, I. E. AND D. SKANDALIS (forthcoming): "Unemployment insurance and job search behavior," *Quarterly Journal of Economics*.

MEYER, B. D. (1990): "Unemployment Insurance and Unemployment Spells," *Econometrica*, 58, 757–782.

MINIMUM WAGE COMMISSION (2018): "Second Report on the Effects of the Minimum Wage," *Berlin*.

MITMAN, K. AND S. RABINOVICH (2019): "Do Unemployment Benefit Extensions Explain the Emergence of Jobless Recoveries?" .

MORTENSEN, D. T. AND C. A. PISSARIDES (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *The review of economic studies*, 61, 397–415.

PORTUGAL, P. AND A. R. CARDOSO (2006): "Disentangling the Minimum Wage Puzzle: An Analysis of Worker Accessions and Separations," *Journal of the European Economic Association*, 4, 988–1013.

POSTEL-VINAY, F. AND J.-M. ROBIN (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–2350.

SCHMIEDER, J. F., T. VON WACHTER, AND S. BENDER (2012): "The Long-Term Effects of UI Extensions on Employment," *American Economic Review*, 102, 514–19.

——— (2016): "The Effect of Unemployment Benefits and Nonemployment Durations on Wages," *American Economic Review*, 106, 739–77.

SHEPHARD, A. (2017): "Equilibrium Search and Tax Credit Reform," *International Economic Review*, 58, 1047–1088.

VAN DEN BERG, G. J. AND G. RIDDER (1998): "An Empirical Equilibrium Search Model of the Labor Market," *Econometrica*, 66, 1183–1221.

VOM BERGE, P., S. KAIMER, S. COPESTAKE, J. EBERLE, AND T. HAEPP (2016): "Arbeitsmarktspiegel: Entwicklungen nach Einführung des Mindestlohns (Ausgabe 2)," Tech. rep., IAB-Forschungsbericht.

A Additional Tables and Figures

A.1 Estimation and Model Fit

TABLE A.1: Model Fit – Job-to-Job Transitions

	full-time	part-time	mini-job
Job-to-job transition			
Data	0.028	0.034	0.088
Model	0.039	0.046	0.062
Godfather shock			
Data	0.017	0.022	0.065
Model	0.017	0.022	0.050

Note: This table shows the probability of job-to-job transitions for full-time, part-time and mini-job workers. The top panel shows the probability of any job-to-job transition and the bottom panel shows the probability of being hit by the Godfather shock. Data: SIAB.

TABLE A.2: Model Fit – Other Moments

	Model	Data
Job Vacancy Rate		
Job Vacancy Rate	0.034	0.025
Job Vacancy Rate (full-time)	0.029	–
Job Vacancy Rate (part-time)	0.040	–
Job Vacancy Rate (mini-job)	0.054	–
Firm Size Distribution		
Mean of log firm size	3.745	4.136
Std. dev. of log firm size	0.928	2.187
Mean of log firm size (full-time)	3.550	4.147
Std. dev. of log firm size (full-time)	1.038	2.173
Mean of log firm size (part-time)	2.199	2.976
Std. dev. of log firm size (part-time)	0.753	2.039
Mean of log firm size (mini-job)	1.132	1.927
Std. dev. of log firm size (mini-job)	0.130	1.707

Note: This table shows . . . Data: BHP, own calculations.

TABLE A.3: Model Fit – Employment Status

	$\Pr(e_f\|e)$	$\Pr(e_p\|e)$	$\Pr(e_m\|e)$	$\Pr(u)$	$\Pr(lu\|u)$
Men, Single					
Data	0.832	0.100	0.068	0.109	0.526
Model	0.849	0.087	0.065	0.079	0.556
Men, Married					
Data	0.908	0.059	0.033	0.040	0.454
Model	0.900	0.078	0.022	0.053	0.482
Women, Single, No Kids					
Data	0.666	0.224	0.110	0.068	0.520
Model	0.697	0.193	0.110	0.081	0.526
Women, Single, Kids					
Data	0.330	0.534	0.136	0.140	0.552
Model	0.230	0.629	0.142	0.104	0.528
Women, Married					
Data	0.309	0.516	0.176	0.040	0.554
Model	0.195	0.633	0.172	0.085	0.488
Total					
Data	0.663	0.240	0.096	0.064	0.518
Model	0.636	0.273	0.091	0.074	0.512

Note: This table shows the share of full-time, part-time and marginal jobs conditional on employment (columns 2-4), the unemployment rate (column 5) and the share of long-term unemployment conditional on unemployment (column 6) for each sociodemographic worker type and in the population (last panel). Data: SIAB.

TABLE A.4: Model Fit – Job finding Probabilities

	$\Pr(e'\|su)$	$\Pr(e'\|lu)$	$\Pr(e'\|e)$
Men, Single			
Data	0.286	0.062	–
Model	0.236	0.060	–
Men, Married			
Data	0.321	0.074	–
Model	0.306	0.080	–
Women, Single, No Kids			
Data	0.321	0.065	–
Model	0.255	0.067	–
Women, Single, Kids			
Data	0.303	0.082	–
Model	0.258	0.067	–
Women, Married			
Data	0.263	0.059	–
Model	0.322	0.078	–
Total			
Data	0.296	0.067	0.035
Model	0.285	0.071	0.043

Note: This table shows the probability of finding a job out of short- and long-term unemployment as well as the job-to-job transition probability for each sociodemographic worker type and in the population (last panel). Data: SIAB.

TABLE A.5: Model Fit – Job Types and Sociodemographics by Wage Groups

	[0, 5.5)		[5.5, 8.5)		[8.5, 12.5)		[12.5, 20)		[20, ∞)		Total	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Job Types												
Full-Time	0.028	0.269	0.358	0.331	0.519	0.558	0.678	0.708	0.766	0.802	0.636	0.663
Part-Time	0.000	0.239	0.290	0.257	0.332	0.289	0.277	0.262	0.229	0.191	0.273	0.240
Marginal	0.972	0.492	0.352	0.412	0.149	0.152	0.045	0.030	0.005	0.007	0.091	0.097
Gender												
Men	0.253	0.312	0.382	0.302	0.438	0.416	0.519	0.519	0.621	0.642	0.520	0.517
Women	0.747	0.688	0.618	0.698	0.562	0.584	0.481	0.481	0.379	0.358	0.480	0.483
Sociodemographics												
Men, Single	0.154	–	0.163	–	0.173	–	0.211	–	0.256	–	0.213	–
Men, Married	0.099	–	0.219	–	0.265	–	0.308	–	0.364	–	0.307	–
Women, Single, No Kids	0.229	–	0.196	–	0.182	–	0.172	–	0.142	–	0.167	–
Women, Single, Kids	0.063	–	0.060	–	0.054	–	0.043	–	0.035	–	0.045	–
Women, Married	0.455	–	0.363	–	0.326	–	0.266	–	0.202	–	0.269	–

Note: This table shows the distribution of job-types, gender and sociodemographics within different parts of the wage distribution. Data: SIAB.

TABLE A.6: Model Fit – Wage Groups by Job Types and Sociodemographics

	[0, 5.5)		[5.5, 8.5)		[8.5, 12.5)		[12.5, 20)		[20, ∞)	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Job Types										
Full-Time	0.000	0.007	0.057	0.048	0.180	0.156	0.358	0.367	0.404	0.422
Part-Time	0.000	0.018	0.108	0.103	0.269	0.224	0.342	0.376	0.282	0.279
Marginal	0.066	0.097	0.393	0.442	0.358	0.317	0.165	0.117	0.019	0.027
Gender										
Men	0.003	0.011	0.075	0.056	0.186	0.150	0.336	0.347	0.401	0.436
Women	0.010	0.025	0.131	0.140	0.258	0.227	0.336	0.346	0.265	0.262
Sociodemographics										
Men, Single	0.004	–	0.078	–	0.180	–	0.334	–	0.404	–
Men, Married	0.002	–	0.073	–	0.190	–	0.337	–	0.398	–
Women, Single, No Kids	0.008	–	0.119	–	0.240	–	0.346	–	0.286	–
Women, Single, Kids	0.009	–	0.136	–	0.265	–	0.328	–	0.262	–
Women, Married	0.010	–	0.138	–	0.267	–	0.332	–	0.252	–
Total	0.006	0.018	0.102	0.097	0.220	0.187	0.336	0.347	0.336	0.352

Note: This table shows the share of workeres in different wage groups conditional on job types, gender and sociodemographics. Data: SIAB.

TABLE A.7: Model Fit – Worker Clustered AKM Fixed Effects

	Total		Men		Women	
	Model	Data	Model	Data	Model	Data
P05 / P50	0.507	0.622	0.507	0.640	0.551	0.601
P10 / P50	0.570	0.687	0.570	0.699	0.619	0.677
P20 / P50	0.690	0.778	0.690	0.783	0.717	0.769
P30 / P50	0.783	0.854	0.783	0.854	0.816	0.850
P40 / P50	0.884	0.925	0.884	0.924	0.922	0.926
P60 / P50	1.090	1.088	1.141	1.092	1.133	1.080
P70 / P50	1.259	1.203	1.329	1.215	1.300	1.178
P80 / P50	1.506	1.370	1.506	1.393	1.444	1.314
P90 / P50	1.777	1.651	1.777	1.653	1.763	1.546
P95 / P50	1.996	1.864	1.996	1.884	2.168	1.760

Note: This table shows the median and selected percentile ratios of AKM worker fixed effects for full-time jobs. Data: SIAB.

TABLE A.8: Model Fit – Firm Clustered AKM Fixed Effects

	Full-Time		Part-Time		Marginal	
	Model	Data	Model	Data	Model	Data
P50 / P50$_f$	1.000	1.000	0.931	0.993	0.823	0.851
P05 / P50	0.798	0.702	0.789	0.689	0.868	0.683
P10 / P50	0.851	0.762	0.811	0.784	0.868	0.789
P25 / P50	0.931	0.877	0.883	0.905	0.918	0.915
P75 / P50	1.145	1.084	1.074	1.125	1.132	1.231
P90 / P50	1.145	1.171	1.229	1.152	1.216	1.312
P95 / P50	1.145	1.171	1.229	1.245	1.392	1.344

Note: This table shows the median and selected percentile ratios of (full-time) firm productivity for full-time, part-time and marginal jobs. The full-time firm productivity is the exponential of the AKM firm fixed effects estimated on wages of full-time workers only. Data: SIAB.

Table A.9: Model Fit – Wages

	Total		Full-Time		Part-Time		Marginal		Men		Women	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Mean	18.11	18.54	19.87	20.38	16.85	9.23	9.65	9.23	19.68	20.78	16.42	16.13
Variance (logs)	0.49	0.51	0.46	0.47	0.46	0.48	0.35	0.37	0.49	0.50	0.48	0.49
P01	5.81	5.01	6.65	5.92	6.53	5.04	4.76	4.30	6.22	5.41	5.56	4.80
P05	7.27	6.79	8.44	8.37	7.61	6.81	5.34	4.97	7.80	7.90	6.92	6.24
P10	8.41	8.18	9.67	9.92	8.35	8.11	5.82	5.51	9.06	9.60	7.89	7.35
P15	9.43	9.32	10.90	11.21	9.15	9.13	6.21	5.92	10.21	11.00	8.70	8.25
P20	10.23	10.42	11.80	12.33	9.99	10.06	6.60	6.27	11.25	12.18	9.50	9.09
P30	12.00	12.52	13.57	14.47	11.32	11.79	7.37	6.92	13.44	14.41	11.00	10.82
P40	14.00	14.63	15.75	16.42	12.96	13.52	8.02	7.55	15.30	16.44	12.52	12.69
P50	15.96	16.67	17.70	18.36	14.77	15.30	8.85	8.20	17.39	18.56	14.26	14.70
P70	21.35	21.48	23.36	23.32	19.51	19.53	10.73	9.86	23.36	23.99	18.95	19.03
P90	31.18	31.66	33.20	33.72	28.46	27.59	14.34	13.86	33.23	34.77	28.29	26.84
P95	36.34	36.00	39.76	37.27	33.37	33.28	16.63	16.86	40.35	39.19	33.21	31.88

Note: This table shows the mean wage, variance and selected percentile of hourly wages in the data and the estimated model. The variance is taken over the log wages. The moments for men and women were targeted in the estimation. Data: SIAB.

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A.2   Results

Table A.10: Minimum Wage Effects – Mechanisms

	(1) Baseline	(2) GE	(3) Fix Search	(4) Fix Vacancies	(5) Fix Surplus	(6) Fix Vac. Shares	(7) Fix Vac. Mass
	Value	Change	Change	Change	Change	Change	Change
Unemployment							
Unemployment Rate	7.44%	-0.035	0.041	-0.086	0.125	0.058	-0.181
$\Pr(e|u)$	17.54%	-0.124	-0.263	0.212	-0.481	-0.174	0.299
$\Pr(su|e)$	1.41%	-0.017	-0.013	-0.001	-0.014	-0.002	-0.014
Employment Level							
Log Hours	3.389	0.014	0.011	0.001	0.012	0.002	0.011
Full-Time Share	63.60%	0.394	0.191	0.023	0.239	0.101	0.370
Part-Time Share	27.26%	0.812	0.808	0.023	0.792	0.009	0.553
Marginal Share	9.14%	-1.206	-0.999	-0.047	-1.032	-0.111	-0.923
Wages, Earnings & Incomes							
Log Wages	2.776	0.021	0.022	0.018	0.022	0.019	0.020
Log Productivity	0.382	0.005	0.008	-0.002	0.008	-0.001	0.006
Log Earnings	7.631	0.035	0.033	0.018	0.034	0.020	0.031
Log Net Earnings	7.279	0.028	0.027	0.015	0.028	0.017	0.025
Log Income	7.583	0.008	0.008	0.004	0.008	0.005	0.007
Macro Aggregates							
Log Output	8.305	0.005	0.005	-0.000	0.005	-0.001	0.005
Log Transfers	4.554	-0.060	-0.048	-0.034	-0.042	-0.026	-0.066
Log Labor Taxes	6.719	0.009	0.008	0.005	0.008	0.005	0.009

Note: This table shows the long-run effect of the introduction of a federal minimum wage of 8.5 EUR relative to the baseline equilibrium without a minimum wage (first column). The second columns shows the new general equilibrium. In the remaining columns, different margins of adjustment are switched off one at a time. In column 3, workers' search effort is held fixed. In column 4, firms' vacancy posting is held fixed. In column 5, workers can adjust their search effort, but the expected surplus of meeting a firm is held constant (search effort will still react to changes in labor market tightness). In column 6, the hours-and productivity shares of posted vacancies are held fixed, but the total mass of vacancies is allowed to adjust. In column 7, the total mass of vacancies is held fixed, but the hours- and productivity distribution of vacancies is allowed to adjust. In all scenarios, unprofitable vacancies may not be posted. Changes refer to the absolute difference to the baseline outcome (e.g. percentage points or log points).

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B   Data and Target Moments

In this appendix, I detail how and with what data the target moments are computed.

B.1   Data

Add description of SIAB and SOEP data!

B.2   Targeted Moments

Sociodemographics   The distribution of sociodemographic types conditional on gender $\Pr(j|g)$ is taken from the SOEP. The distribution of gender $\Pr(g)$ is taken from the SIAB in order to use as much administrative information as possible.

Labor Market States   As the SIAB data does not contain sociodemographic information for employed workers, I have to fill some gaps with information from the SOEP while ensuring that the joint distribution of gender and labor market status remains consistent with the administrative SIAB data.

I start by computing the unemployment rate conditional on j such that it is consistent with the gender-specific unemployment rate in the SIAB

$$\Pr(u|j) = \Pr(u|j,g) = \frac{\Pr(u,j|g)}{\Pr(j|g)} = \frac{\Pr(j|u,g)\Pr(u|g)}{\Pr(j|g)} \tag{B.1}$$

where only $\Pr(j|g)$ is taken from the SOEP. I use blue font color to highlight moments taken from the SOEP. The probability of long-term unemployment conditional, $\Pr(lu|u,j)$, is taken directly from the SIAB.

Computing the share of type-j workers who have a type-x job requires slightly more information from the SOEP:

$$\Pr(e_x|j) = \Pr(e_x|j,g) = \frac{\Pr(j|e_x,g)\Pr(e_x|g)}{\Pr(j|g)} \tag{B.2}$$

Transition Probabilities   The job finding rate out of short- and long-term unemployment, $\Pr(e'|su,j)$ and $\Pr(e'|lu,j)$, can be computed using SIAB data only. As I do not target job-to-job transition probabilities by sociodemographics, they are computed as the share of workers who change their employer or job type.

Hourly Wage Quantiles   To compute hourly wages based on daily earnings reported in the SIAB data, I impute average hours worked per day using data from the SOEP and job-type dependent averages reported by Dustmann et al. (2020) who have confidential information on hours for the social security data in 2014.

The average adjusted hours for full- and part-time jobs in Dustmann et al. (2020) are almost identical to the averages in the SOEP and Structure of Earnings Survey (SES).[35] The only dif-

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[35]Dustmann et al. (2020) adjust the raw contracted working hours in the social security records to account for differences in whether sick leave and overtime are included in the contractual hours.

ference between the three data sets is that, for mini-jobs, average hours worked are substantially higher in the SOEP.

For full-time jobs, I set daily hours to 7.8 which corresponds to 39 hours per week. For par-time and mini-jobs, I use the joint distribution of hours and earnings from the SOEP to take into account that some of the variation in earnings is driven by heterogeneity in hours worked. To that end, I compute the mean and standard deviation of contractual hours worked within different earnings bins. I then draw hours worked from a Normal distribution with these parameters and impose that weekly hours for part-time and mini-jobs be in the interval $[5, 35]$ and $[2, 20]$ respectively.[36] Finally I rescale the hours worked such that, on average, part-time employees work 24 hours and mini-job employees 8.7 hours per week – as reported in Dustmann et al. (2020).

Hourly wages are then computed as earnings divided by imputed hours worked. I target the 0.01, 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.7, 0.9, and 0.95 quantiles of the wage distributions conditional on job type and conditional on gender (separately). In addition, I target the share of part-time and mini-jobs and the share of men in the following five wage groups $(0, 6.5)$, $[6.5, 8.5)$, $[8.5, 12.5)$, $[12.5, 20)$, $[20, \infty)$.

Worker and Firm Fixed Effects In the absence of a minimum wage, the wage equation in my model is very simple. As in Abowd et al. (1999) (henceforth AKM), the wage w of a full-time worker employed at firm with productivity p is log-additive in her skill h and the firm's productivity

$$\log(w) = \log(r) + \log(h) + \log(p) \tag{B.3}$$

where r is the exogenous piece-rate. I estimate the empirical distribution of worker and firm-class fixed effects using a clustered AKM approach (Bonhomme et al., 2019).

While the model is consistent with an AKM-style wage equation (Abowd et al., 1999; Card et al., 2013), I do not estimate the model by straight AKM because of two distinct reasons. First, while the SIAB data is large compared to survey data sets, it covers only 2% of all workers and the firms they are employed at. This implies that the connected set of firms and workers on which firm and worker fixed effects can be identified is too small. Second, estimation would suffer from severe incidental parameter bias as the number of movers between two firms tends to be low.

Instead, I estimate the empirical distributions of worker and firm heterogeneity using the approach recently proposed by Bonhomme et al. (2019) (henceforth BLM) which solves both of these issues using dimension reduction techniques. The proposed method is particularly useful as it can be applied to data sets that cover only few firm-to-firm moves. The key assumption is that unobserved firm heterogeneity operates on the level of discrete firm classes rather than on the level of individual firms. Given an estimated partition of all firms into classes, firm class and worker fixed effects are identified from job-to-job transitions between firms of different classes

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[36]For part-time jobs, I use 500, 750, 1000, 1500, ..., 4000, 5000, 10000 Euro as cutoffs to define the monthly earnings bins. For mini-jobs, I use the cutoffs 100, 150, ..., 500 Euro.

rather than between different firms. This allows estimation of worker and firm (class) effects on much smaller samples of linked employer-employee data such as the SIAB (2%).

Class membership is estimated using K-means clustering that minimizes the within-class variation of within-firm earnings distributions:

$$\min_{k(1),...,k(J),H_1,...,H_K} \sum_{j=1}^{J} \frac{1}{M} \sum_{m=1}^{M} \left(F_j^m - H_{k(j)}^m\right)^2$$

where $k(j)$ is the class of firm j, F_j^m is an observable characteristic of firm j and H_k^m is the average of that characteristic across all firms in class k. I classify firms based on information on the within-firm wage distribution. In particular, I use the mean, selected percentiles (25, 50, 75) and the share of workers with top-coded earnings for full-time employees.[37] Consistent with the model where firm productivity is deterministic, I average these characteristics at the firm level over the years 2011 to 2014. This yields a time-invariant classification of firms.

Given the firm classification, I estimate the worker and firm-class fixed effects conditional on the estimated, i.e. run a clustered AKM estimation without covariates (except time fixed effects).

$$\log(w_{it}) = \alpha_i + \psi_{k(j(it))} + \gamma_t + \varepsilon_{it} \tag{B.4}$$

I then target the distribution of α conditional on gender and the worker-weighted distribution of ψ to inform the distributions of human capital and firm productivity. In particular, I target the quantile ratios $q_x^k/q_x^{0.5}$ for $k = 0.01, 0.05, 0.1, 0.3, 0.7, 0.9, 0.95, 0.99$ and $x \in \{f, p, m\}$, where q_x^k is the k-quantile of the distribution of ψ weighted by the firm's number type-x workers. In addition, I target $q_x^{0.5}/q_f^{0.5}$ for $x \in \{p, m\}$. Finally, I target the shares of the variance of log wages explained by the worker and firm components as well as the correlation between worker and firm fixed effects.

Firm Size   The mean and standard deviation of log firm size are computed using administrative data from the Establishment History Panel. For consistency with the worker moments, I only consider employees between 25 and 60 years of age and drop firms that do not have employees in this age range.

Job Vacancy Rate   The job vacancy rate is the number of vacancies relative to the sum of vacancies and jobs. As many vacancies are not officially registered, I do not rely on the job vacancy rate reported by Eurostat but rather use the Job Vacancy Survey (JVS).[38] The JVS contains both registered and unregistered vacancies – each account for roughly half of the total number of vacancies. In 2014, around 900 thousand vacancies were open. With roughly 36 million jobs, this gives a job vacancy rate of 2.44%.[39]

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[37]This information is made available for every firm such that the within-firm earnings distribution can be approximated without observing a representative sample of employees for each firm.

[38]See Brenzel et al. (2016) for details about the data.

[39]Source: Statistics of the Federal Employment Agency.

C   Computational Details

C.1   Steady State

To be added!

C.2   Transition Path

Starting from the terminal stationary equilibrium, I guess a path for all equilibrium objects and solve backwards. We focus on one generic skill segment h and drop h form the notation to improve readability.

Firm Problem   Assuming that the new stationary equilibrium is reached after T periods, the firm's expected value of an employment relationship with a type-j worker starting in period t is:

$$W_t^j(x,p) = y + \sum_{s=t+1}^{T-1} \beta^{s-t} \underbrace{\prod_{k=t}^{s-1}(1 - \delta_k^j(x,p))}_{\Pr(\text{survival until } s)} y(x,p) + \beta^T \underbrace{\prod_{k=t}^{T-1}(1 - \delta_k^j(x,p))}_{\Pr(\text{survival until } T)} W_T^j(x,p)$$

$$\text{with}\quad W_T^j(x,p) \equiv W^{j*}(x,p) = \frac{y(x,p)}{1 - \beta(1 - \delta^{j*}(x,p))}$$

Given $W_{t+1}^j(x,p)$ and taking as given the vacancy filling rates $\eta_t^j(x,p)$, the firm optimally chooses the number of vacancies $v_t(x,p)$ to post in period t. Optimal vacancies

$$\kappa'(v_t(x,p),x,p) = \beta^f \sum_j \eta_t^j(x,p) W_{t+1}^j(x,p)$$

$$= \beta^f \Pi(\theta_t) \frac{S_t(x,p)}{S_t} \sum_j \frac{S_t^j(x,p)}{S_t(x,p)} W_{t+1}^j(x,p) \tag{C.5}$$

The firm's optimal policy thus depends on the workers' search policies and distribution over labor market states via $S_t(x,p)$, S_t and $S_t^j(x,p)$. It also depends on θ_t which is a function of the other firms' policies and S_t.

Worker Problem   Workers take as given next period's value functions V_{t+1} – and hence the expected surplus of finding a job – as well as the job filling rate $\Lambda(\theta_t)$ and vacancy shares $N_t(x,p)/N_t$ and choose their optimal search effort according to the resulting first order condition.

$$\frac{\mathrm{d}d^j(\ell)}{\mathrm{d}\ell} = \beta \phi_s \Lambda(\theta_t) \left(\sum_{x,p} \frac{N_t(x,p)}{N_t} \max\left\{ V_{e,t+1}^j(x,p), V_{s,t+1}^j(x,p) \right\} - V_{s,t+1}^j(x,p) \right) \tag{C.6}$$

The workers' optimal policies thus depends on the firms' vacancy policies and via $N_t(x,p)$ and N_t. It also depends on θ_t which is a function of the other workers' policies and N_t.

Algorithm   Focus on one skill segment h and let F_t be the distribution of workers across labor market states in period $t = 0, \ldots, T$. The economy is in the initial regime until period $t = -1$.

We thus set F_0 equal to the stationary distribution in the initial regime. We assume that the economy has converged to the new regime by period T. All equilibrium objects in period T are thus the equilibrium objects in the stationary equilibrium. The main backward looking object is F_t. Search mass, vacancy mass and tightness can be adjusted instantly and are thus allowed to jump from $t = 0$ to $t = 1$. The distribution F_t only jumps due to non-employability.

Knowing the initial and terminal stationary equilibrium, we proceed as follows.

1. Guess a sequence $\{F_t^0\}_t$, e.g. a piece-wise linear interpolation between F_T and F_0 taking into account the employability constraint.

2. Set $i = 0$

3. Taking as given the sequence of distributions $\{F_t^i\}_t$ as well as the value functions W_T^j and $V_{s,T}^j$, solve backwards for the equilibrium sequence of policies $\{\ell_t^i, v_t^i\}_t$. Starting with $t = T - 1$, we solve for the equilibrium policies in t as follows:

   (a) guess vacancy shares and tightness: $N_t(x, p)$ and θ_t
   (b) solve for optimal search policies $\ell_t^i(j, x, p)$ using equation (C.6)
   (c) update $S_t(x, p)$, S_t, $S_t^j(x, p)$ and θ_t
   (d) solve for optimal vacancy policies $v_t^i(x, p)$ using equation (C.5)
   (e) compute implied vacancy shares and tightness
   (f) if equal to guess, stop, else update guess and go back to (b)
   (g) compute the workers' value: $V_t^j(\sigma) = u^j(\sigma, \ell^i) + \beta \mathbb{E}_{\sigma'|\sigma}\left[V_{t+1}^j(\sigma')|\sigma\right]$
   (h) compute the firm's values: $W_t^j(x, p) = y(x, p) + \beta(1 - \delta_t^j(x, p))W_{t+1}^j(x, p)$

4. Set $t = t - 1$ and and repeat until $t = 0$

5. Use the transition matrices P_t^i to iterate forward on the distribution starting from F_0 until F_T to get $\{F_t^{i+1}\}_t$

6. Check whether the implied sequence $\{F_t^{i+1}\}_t$ differs from the guess $\{F_t^i\}_t$. Stop if yes. Set $i = i + 1$ and go back to step (3)

D   Identification Analysis

The use of Sobol points allows for a straight-forward analysis which parameters are identified by which moments. Appendix D contains a series of plots visualizing the relationships between parameters and moments. For each combination of parameter and moment, I regress the moment on a set of dummies that partition the parameter space that was searched in stage 1. For each parameter, I then rank the moments by the R^2 of these regressions and visualize the relationship between the parameter and the moment for the twelve moments most affected by the parameter.

Details to be added!