Greater search cost reduces prices

Sander Heinsalu*

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Abstract

Consumers know their valuation but not the price for many goods and services. In such markets, the optimal price of each firm falls in the search cost of consumers, eventually to the monopoly level, despite the exit of lower-value consumers when search becomes costlier. The reason is that consumers who switch firms can be held up by charging a high price. Greater search cost reduces the fraction of incoming switchers in each firm's demand, which decreases the hold-up motive, thus the price.

Keywords: Search cost, hold-up, imperfect information, price competition.

JEL classification: D82, C72, D43.

Consumers usually know their valuation for common groceries, gasoline or legally standardised notary certification services, but may not know the current price. In case some customers are uncertain about some aspects of the good or service, industry associations often provide a public member directory[1] that reduces customers' search costs and informs them about the value provided by each firm, but does not reveal prices. This suggests that firms in the

*Research School of Economics, Australian National University, Email: sander.heinsalu@anu.edu.au, The author thanks the seminar audience at MIT, Boston University, the University of Copenhagen and Université de Montréeal for comments and suggestions. The hospitality of MIT during part of this work is greatly appreciated.

[1] Grocers: http://www.agbr.com/store-locator/, notaries: https://www.thenotariessociety.org.uk/notary-search, restoration contractors: https://www.iicrc.org/page/IICRCGlobalLocator.

industry benefit from cheaper search by enough to justify maintaining and updating the searchable directory on the web. Car dealerships, gas stations and department stores co-locate to reduce search costs, as discussed below. Easier comparison of firms seemingly increases competition and reduces profits—an intuition confirmed by most industrial organisation models. By contrast, this paper shows how differentiated firms can in fact increase profits by reducing search costs and making the surplus offered more comparable across firms.

The economic forces can be elucidated in a simple model of a duopoly competing on price, but extend more broadly. Firms simultaneously set prices. Consumers privately know their valuations for both firms, which may be determined by geographic location, brand or the preferred time for a service (flight or other transit schedule is known to frequent travellers, but discounts and sold-out cheapest seats make the price uncertain). Initially, each consumer observes the price one of the firms.[2] The consumer may buy immediately from this firm, exit, or learn the price of the competitor. After learning, consumers may buy from either firm or exit. Firms cannot distinguish customers who buy immediately from those who first learn and then buy.[3]

The consumers who learn can be held up, because their willingness to pay the search cost implies that their valuation for the firm they arrive at is above its equilibrium price. This hold-up motive increases prices. Greater search cost weakens the hold-up motive, because fewer consumers search. When a smaller fraction of a firm's demand is composed of customers switching from the competitor, the hold-up motive and the price are lower.

Another intuition is that higher search costs cause some consumers to exit who previously would have switched firms. More exit leaves fewer inframarginal consumers to firms on average, so some firm's inframarginal demand falls. If the firms are symmetric enough, then each of them receives fewer

[2]The first price observation being free is the standard assumption to avoid a market breakdown resulting from the Diamond paradox.

[3] Websites try to track buyers' browsing history to segment them into switchers and captive customers, but buyers may take countermeasures (using a VPN, the Tor browser, or searching on different devices). Also, the segmentation may be illegal or create negative publicity, making it not worthwhile.

inframarginal buyers. The number of marginal consumers may rise or fall in the search cost, but this change is smaller. As the ratio of inframarginal to marginal consumers falls, so does the optimal price. The prices of the firms are strategic complements, so one firm's price cut motivates others to follow suit.

Literature Search costs and pricing have been extensively studied. The literature mostly finds that prices increase in the search cost, as in the seminal work of Diamond (1971). Exceptions assume either multiproduct or multiperiod markets or add a countervailing force (higher switching cost or firm obfuscation) to a lower search cost. The current work embeds the idea of hold-up in a duopoly search model and presents a simpler one-shot, one-product framework, with a different driving force (reduced hold-up) and a stronger result—prices strictly decrease in the search cost for any positive search cost at which some consumers still switch firms.

In Zhou (2014), multiproduct search makes products complements: a price cut on one increases demand for both, more so at a greater search cost. Thus prices may fall in the search cost. Rhodes and Zhou (2019) extend this result to four firms who supply two products and may merge pairwise into two-product firms. Higher search cost may cause mergers, which may reduce prices due to the complementarity mechanism of Zhou (2014). In the present work, the mechanism is hold-up of arriving searchers instead of the deterrence of further search.

Klemperer (1987, 1995) points out that if consumers have a switching cost after their first purchase, then higher switching costs may reduce prices even below cost in the first period. The reason is that firms compete to lock in customers to later charge the monopoly price. The second-period prices weakly increase in the switching cost. Similar economic forces in Choi et al. (2018) cause prices to fall in the search cost under ordered search when firms advertise prices. Firms compete to be the first to be inspected by a consumer, because the search cost of inspecting another firm locks in the consumer. The larger the search cost, the stronger the lock-in and competition, thus the lower the prices. Instead of the competition to lock in customers who have not searched other firms, the current paper studies the moral hazard of holding up consumers who have already learned about both firms.

Dubé et al. (2009) show numerically and Cabral (2009) analytically that an intermediate switching cost leads to lower prices than a zero switching cost in an infinite horizon model. The incentive to cut price to 'invest' in customer acquisition outweighs the incentive to 'harvest' with a high price. However, for large enough switching costs, prices rise. Cabral (2016) extends these results to show that if trades have high frequency or the market structure is close to symmetric duopoly, then switching costs increase competition, but with infrequent trade or sufficiently asymmetric competitors, switching costs decrease competition. The one-shot model of the present paper focusses on the hold-up of switchers, not on investing in customer acquisition over time, and provides conditions for prices to globally decrease in the search cost.

Moraga-González et al. (2017) assume infinitely many symmetric firms and consumers. Consumers learn the valuations and the prices. Lower search costs make existing consumers search more (the usual intensive margin effect) and attract new consumers with relatively high search costs (the extensive margin). If the density of search costs has a decreasing likelihood ratio, then the extensive margin outweighs the intensive margin, so a second order stochastic dominance increase in search costs reduces the symmetric equilibrium price. The present work shows that the intensive margin effect reverses if consumers know their valuations. Section 3 argues the extensive margin is not needed.

Lal and Sarvary (1999) model a firm which may add a web shop to a physical store. They assume that the web shop reduces search costs but increases switching costs, because it is easy to re-order a familiar brand. This may raise prices and reduce search for some parameter values. The effect is driven by the higher switching cost, which outweighs the lower search cost. In the current paper, the search cost is the only force and raising the search cost from any positive level reduces prices.

In Ellison and Wolitzky (2012), firms obfuscate to increase consumers' search cost. With costless obfuscation, firms exactly offset a fall in the exogenous search cost, so it does not affect prices. Thus it may be said that prices weakly increase in the search cost.

Stiglitz (1979) argues that search cost causes breakdown of a competitive market (no consumer enters because each firm holds up all arriving buyers) and thus monopoly or a cartel is Pareto superior. The key is that consumers have to pay a search cost for their first price observation. If learning the first price was costly in the present work, then a similar breakdown would occur. If Stiglitz (1979) was modified to make the first price observation free, then the Diamond paradox would result (monopoly price for any positive search cost) because firms are identical. By contrast, in the current work, firms are horizontally differentiated, so prices vary from the monopoly level and respond to the search cost.

This paper studies unordered consumer search, unlike the more distantly related articles Armstrong et al. (2009); Armstrong (2017), which focus on different questions: the existence of equilibria, how purchase history, price and non-price advertising affect search, the equivalence of ordered search to discrete choice. An additional difference from the current paper is that consumers know their valuations.

Anderson and Renault (2006) assume a monopolist who chooses whether to advertise product or price information or both. The monopolist prefers conveying no information about consumer valuations to full information, the opposite of the duopolists in the current paper. The duopolists never profit from price disclosure, but the monopolist does for some parameter values. The monopoly price decreases in the search cost, is constant for low and high enough costs. The present work derives a stronger result: both duopoly prices fall in the search cost for any positive cost at which search occurs. In Anderson and Renault (2006), for parameters at which the monopoly price decreases in the search cost, the monopolist advertises the price. The reason for the price reduction is to offset the costlier search and thereby attract consumers to visit the monopolist. Advertising commits the monopolist to a price, eliminating hold-up (the mechanism in the current paper for the duopoly results).

Anderson and Renault (2000) study the negative externality that consumers informed about their valuations impose on the uninformed by making demand more inelastic. Buyers cannot exit. Search cost is paid for all price observations, including the first. Prices strictly increase in the search cost iff the countercumulative valuation distribution is log concave and all consumers are uninformed. Prices also increase in the fraction of informed consumers and if this goes to one, then prices go to infinity. In the current paper, consumers can exit, know their valuations, and the first price observation is free. Prices are always finite and some consumers buy.

Incomplete information is another market friction that may reduce prices, contrary to the usual intuition (Heinsalu, 2020). The reason is downward price signalling (the good type of each firm separates from the bad by setting a lower price) coupled with competition (the bad type firms are in a Bertrand race to the bottom), not hold-up of switchers. Price signalling is also studied in Rhodes (2015), where multiproduct firms advertise one product's price to signal the price of the other. Prices increase in the search cost, diametrically opposite to the present work.

Applications Martin (2020) demonstrates that prices decrease in the search cost in the German gasoline market. Search costs are measured as a decrease in the amount of information the German government comparison database of gas stations provides. Nishida and Remer (2018b) find that for isolated gas stations (those not on the same street intersection) located in California, Florida, New Jersey and Texas, reducing the mean search cost by 20% leads to price increases in 32% of markets and an average price increase of 5.2 cents per gallon. Nishida and Remer (2018a) Table 4 shows that prices are on average slightly lower at isolated gas stations than in the full sample. Isolated stations are costlier to search because consumers have to travel a greater distance from one to another to observe the price.

Car sellers post the technical specifications of cars online and these are credible, but the large price signs on the windscreens or websites are not, because haggling over price is the norm, conveyance, handling and service fees are added and almost universally used accessories such as floor mats and mud guards cost extra. Thus the sellers reveal valuations but not prices to consumers, even though committing to prices is possible. Sellers also reduce consumers' search costs by co-locating: next to a major road there are often many car dealerships in a row. In Murry and Zhou (2020) Table 11, the effect of closing a co-located car dealer is to reduce its competitors' prices by 0.1%. Closure of competitors has the effect of increasing search costs for consumers who now have to travel further from one dealer to another for an additional price observation.

Agglomeration is often explained by the motive to reduce the search costs of consumers. Clusters such as shopping malls compete with each other to attract visitors who prefer a larger cluster for variety or ease of comparison. However, within a cluster, sellers still compete, so should prefer to locate as far from each other as possible. This differs from the observation that in a shopping mall, similar shops locate close to each other (groceries on the ground floor, jewellery, cosmetics and clothes on higher floors). The current paper explains within-cluster agglomeration: prices and profits decrease in the search cost whenever it is positive. Vitorino (2012) Table 8 shows the profits of midscale department stores (e.g., Mervyn's, JC Penney) increase in the number of midscale stores in the mall.

The next section introduces the framework and derives the main result. Extensions and generalisations are discussed in Section 3, followed by the conclusion in Section 4.

1 Horizontally differentiated duopoly

Two symmetric[4] firms $i \in \{X, Y\}$ simultaneously set prices P_i. There is a mass 1 of consumers indexed by $v = (v_X, v_Y)$, where $v_i \in [0, 1]$ is the consumer's valuation for firm i's product. Consumers privately know their valuations. Firms only have the common prior belief that v_i is distributed according to f, which is positive with interval support. The corresponding cdf is denoted F. The valuations v_X, v_Y are independent.[5]

[4] The case of firms with asymmetric costs and valuation distributions is solved in an earlier version of this paper, available on https://sanderheinsalu.com/. The direction of comparative statics is the same.

[5] Section 3 considers correlated valuations and other extensions.

Independently of v, half of the consumers initially observe P_X and half P_Y. Call the firm whose price a consumer initially observes the *initial firm* of the consumer. Each consumer decides whether to buy from her initial firm, learn the price of the other firm at cost $s > 0$[6] or exit. After learning, the consumer decides whether to buy from firm X, firm Y or exit.

The payoff from not buying is normalised to zero. A consumer with valuation v who buys from firm i at price P_i without searching obtains payoff $v_i - P_i$, but after search, obtains $v_i - P_i - s$ from buying and $-s$ from exiting. Firm i that sets price P_i resulting in *ex post* demand D_i gets *ex post* profit $\pi_i := P_i D_i$. The marginal cost is assumed constant and normalised to zero. W.l.o.g. restrict $P_i \in [0, 1]$, because a price that is negative or above the maximal valuation of consumers is never a unique best response. Mixed prices on or off the equilibrium path are ruled out in the appendix in Lemma 7.

Equilibrium consists of the firms' pricing strategies and consumer decisions such that (i) each firm maximises profits given the decisions it expects from the consumers and the rival firm. (ii) Consumers maximise their expected payoff by choosing to buy, learn or exit based on the prices they see and expect. Consumers who learn choose which firm, if any, to buy from to maximise their expected payoff. (iii) The expectations of the firms and consumers are correct.

The next section first finds the optimal decisions of consumers, which determine the demands for the firms. Then the profit-maximising prices are calculated, followed the main comparative static of prices decreasing in the search cost.

2 Demand, profit and comparative statics

To solve the pricing game, start with the decisions of the consumers. These determine the demands for the firms, which are then used to find the optimal prices.

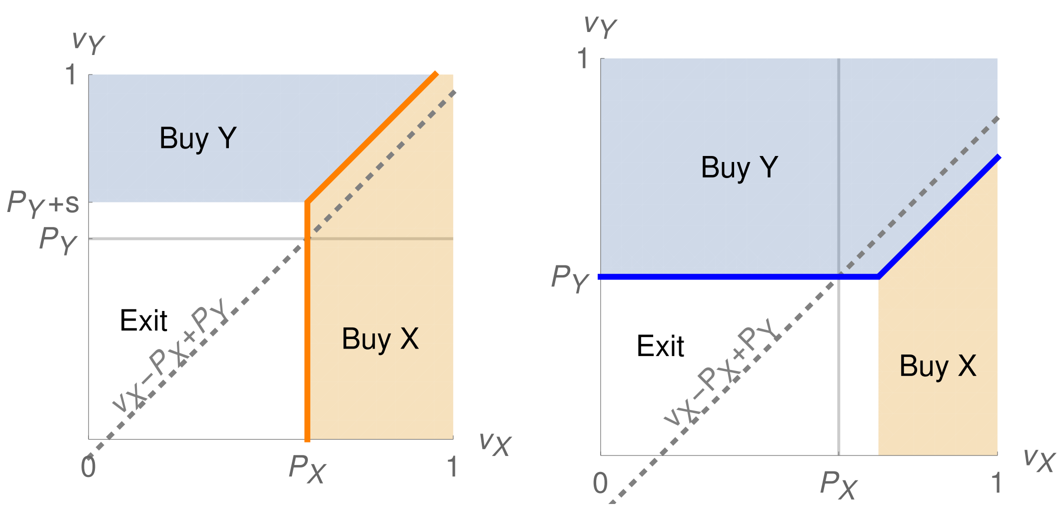
Consumer v who observes firm j's price P_j and expects firm i to choose $P_i^{\mathbb{E}}$

[6] Zero search cost is qualitatively different (Bertrand competition). Section 3 discusses $s = 0$.

learns P_i if $\max\left\{0, v_i - P_i^{\mathbb{E}}, v_j - P_j\right\} - s \geq \max\left\{0, v_j - P_j\right\}$. The right-hand side (RHS) is the value of not learning—either choosing to exit (payoff zero) or to buy immediately at price P_j. The left-hand side (LHS) is the benefit of learning minus the search cost s. The benefit includes the options of being able to exit, buy from firm i or buy from firm j after learning. The consumer chooses the best of these options.

The demand for a firm consists of consumers initially at that firm who either buy immediately or learn and then buy from that firm, and consumers initially at the rival firm who learn and switch. Figure 1 depicts demands for each firm from customers initially at each firm (left panel: buyers initially at firm X, right panel: Y). The marginal customers for firm Y are marked as the thick blue line and the marginal buyers for X as the thick orange line. Consumers initially at X are not marginal for Y and vice versa.

Figure 1: Demands at the pure prices $P_X = P_X^* = P_{XCE} = 0.6$ and $P_Y = P_Y^* = P_{YCE} = 0.45$ and search cost $s = 0.1$. Left panel: consumers initially at firm X, right panel: Y.



Customers initially at firm i buy immediately from i if $v_i - P_i \geq \max\left\{0, v_j - P_j^{\mathbb{E}} - s\right\}$. They learn and then buy from firm i when both $v_j - P_j^{\mathbb{E}} - s \geq \max\left\{0, v_i - P_i\right\}$ and $v_i - P_i \geq \max\left\{0, v_j - P_j\right\}$. These conditions can be combined to $v_i \geq P_i + \max\{0, v_j - \max\{P_j^{\mathbb{E}} + s, P_j\}\}$.

Consumers starting at j buy from i if both their expected payoff from learning $v_i - P_i^{\mathbb{E}} - s$ and the observed payoff $v_i - P_i$ from i after learning are larger than the payoff $\max\{0, v_j - P_j\}$ from exiting or buying from j. Combining these conditions results in $v_i \geq \max\left\{P_i, P_i^{\mathbb{E}} + s\right\} + \max\{0, v_j - P_j\}$.

The demand that firm i expects from price P_i when it expects firm j to choose P_j^*, consumers initially at j to expect $P_i^{\mathbb{E}}$ and consumers at i to expect $P_j^{\mathbb{E}}$ is

$$\begin{aligned} D_i(P_i, P_j^*, P_i^{\mathbb{E}}, P_j^{\mathbb{E}}) = &\frac{1}{2}\int_0^1 \int_{P_i+\max\left\{0, v_j - \max\left\{P_j^{\mathbb{E}}+s, P_j^*\right\}\right\}}^1 f(v_i)f(v_j)dv_i dv_j \\ &+ \frac{1}{2}\int_0^1 \int_{\max\left\{P_i, P_i^{\mathbb{E}}+s\right\}+\max\left\{0, v_j - P_j^*\right\}}^1 f(v_i)f(v_j)dv_i dv_j. \end{aligned} \quad (1)$$

The two integrals in the demand aggregate the consumers initially at each firm over the region of valuations that result in these consumers eventually buying from i, given the prices.

For a uniform valuation distribution, the equilibrium prices can be found analytically: $P_i = P_j = \frac{-1-s+\sqrt{5+2s-s^2}}{2}$. Prices are strictly decreasing strictly concave in the search cost for all $s \in [0,1]$. Subsequent results use a general valuation distribution f.

Having derived the demand, the next lemma establishes the strategic complementarity of prices. A sufficient condition is that the densities of consumer valuations do not decrease too fast. The uniform and truncated exponential distributions satisfy the condition, as does any increasing density.

Lemma 1. *If $P_i \frac{\partial f(P_i+w)}{\partial P_i} \geq -f(P_i + w)$ for all $P_i \in (0,1)$ and $w \in [0, 1-P_i)$ and for each firm i, then prices are strategic complements.*

The proofs of all results are in the appendix.

The conditions in Lemma 1 are sufficient but not necessary for prices to be strategic complements. With strategic complementarities, the prices of the firms move together, the equilibria with the lowest and highest prices are stable, and all stable equilibria have the same comparative statics. A firm's profit increases in a rival's price, so firms impose positive externalities on each other by raising price. This implies that for the firms, equilibria are Pareto ordered by price. The highest-price equilibrium is the natural focus of coordination if multiple equilibria exist.

The next lemma shows that equilibrium is unique if the consumer valuation pdf is weakly decreasing but not too fast (e.g., uniform).

Lemma 2. *If for each firm,* $\frac{\partial f(P_i)}{\partial P_i} \leq 0$ *and* $P_i \frac{\partial f(P_i)}{\partial P_i} \geq -2f(P_i)$ *for any* $P_i \in (0, 1)$*, then the equilibrium is unique.*

The conditions in Lemma 2 are sufficient, but not necessary for uniqueness. In addition, uniqueness is not necessary for the main result because strategic complementarity makes the direction of comparative statics of all stable equilibria the same.

The main theorem establishes that if the consumer valuation distribution does not vary too fast, then each firm's price decreases in the search cost of the consumers. Uniform valuations satisfy the condition, as does a truncated exponential distribution if search is not too costly.

Theorem 3. *If* $f(P_i - s + w) + P_i \frac{\partial f(P_i - s + w)}{\partial P_i} \leq f(P_i + s + w)$ *for all* $P_i \in (0, 1)$*,* $w \in [0, 1 - P_i + s)$*, then* $\frac{dP_i^*}{ds} \leq 0$ *for both firms in any stable equilibrium. If further* $s < 1$*, then* $\frac{dP_i^*}{ds} < 0$*.*

The intuition for Theorem 3 is that the fraction of switchers among a firm's customers falls in the search cost. The switchers can be held up, because they are willing to pay the price plus the search cost, thus will all still buy if the firm's chosen price exceeds the expected price slightly. This hold-up motive increases a firm's optimal price. Greater search cost decreases the hold-up motive, thus the price. When the search cost becomes so large that no consumers switch, each firm's price falls to its monopoly level. This monopoly price is with respect to the remaining demand at the large search cost when low-valuation customers have exited. This demand is smaller than at lower search costs and contains relatively more high-valuation customers. Therefore the monopoly price at the remaining demand is greater than for a joint owner of the firms at a smaller search cost.

Figure 2: Demands after an increase in the search cost from 0.1 to 0.2 at $P_X = 0.6$, $P_Y = 0.45$.

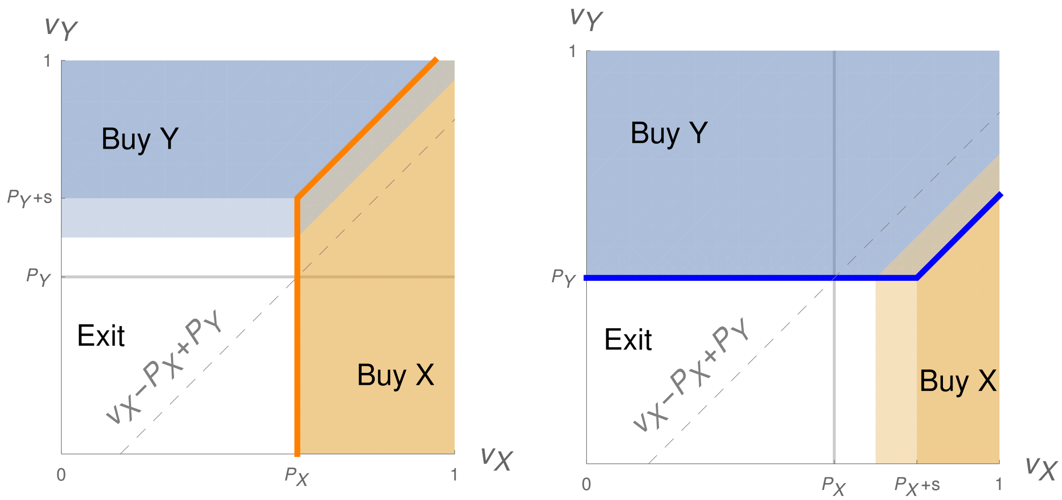


Figure 2 shows the effect of a greater search cost on the demands, fixing the prices. In the left panel (consumers initially at firm X), the light blue rectangle is the consumers who stop buying from Y and exit when s increases. The bluish diagonal band below the orange line consists of the consumers who start buying from X instead of Y. In the right panel, the orange rectangle shows the consumers who stop buying from X and exit, while the orange diagonal band above the blue line depicts those who stay with Y instead of switching to X. Compared to before the search cost increase, each firm loses some switchers who could be held up and gains some demand from consumers initially at itself. The latter respond to any price increase, however small. With a uniform valuation distribution and symmetric initial demands, firm X gains marginal consumers and firm Y loses a small measure, but both firms lose a substantial mass of inframarginal customers.

This concludes the discussion of how the search cost affects prices. The following subsection examines the change of profit, welfare and consumer surplus in the search cost, as well as how prices respond to the initial allocation of consumers.

2.1 Other comparative statics

Exit increases in the search cost, so total surplus (buyer value minus seller cost integrated over all trading buyers and sellers) in the market falls. Conditional on the consumers who trade, price is a transfer that does not affect total surplus, just profits and consumer surplus. Costlier search also makes the final allocation of the buyers to the firms less efficient (ideally, consumers above the diagonal in Figure 2 would buy from firm Y and below the diagonal from X). Therefore welfare and profits decrease in the search cost. The reasoning above is formalised in the following result.

Proposition 4. *If $f(P_i) + P_i \frac{\partial f(P_i)}{\partial P_i} \geq 0$ for all P_i, then in any equilibrium, total surplus and each firm's profit and demand decrease in the search cost.*

The effect on consumer surplus could have either sign in general, because both prices and allocative efficiency decrease. With a uniform valuation distribution, consumer surplus strictly decreases in the search cost.

Suppose all consumers are initially at one firm (the incumbent) instead of a half-half split. Then the unique equilibrium prices are such that the incumbent remains a monopolist. The consumers expect a high enough price from the other firm (the entrant) that learning is not optimal. Any price cut by the entrant is not observed by consumers, so they cannot start learning in response to it. Suppose consumers expected the entrant to set a low enough price to make learning worthwhile at some valuations. Then the entrant would hold up all arriving switchers by choosing a price greater than they expected, for any expected price and any positive search cost. This contradicts consumers learning the entrant's price.

Modifications of the baseline model are considered next. The results remain robust to a distribution of search costs, unattached consumers or many firms, and are continuous in the correlation of valuations.

3 Extensions and generalisations

With zero search cost, prices are discretely lower than with a small positive search cost, because the mass of inframarginal consumers is continuous in s everywhere, including at $s = 0$, but the mass of marginal consumers approximately doubles at $s = 0$ compared to a small positive s. Hold-up is impossible if consumers can costlessly switch firms. The discontinuity in prices at zero search cost is similar to the Diamond paradox.

If fraction α of consumers are 'shoppers' who have zero search cost and $1-\alpha$ are captive or loyal with $s > 0$, then demand for firm i is $\alpha \iint_{v_i \geq P_i + \max\{0, v_j - P_j\}} dF^2(v_i, v_j) + (1-\alpha)D_i$, where D_i is the demand of the captive customers defined in (1). Because firms are horizontally differentiated, a Bertrand outcome is avoided even when all consumers are shoppers. The mass of marginal consumers increases in α for each firm at any price combination, but the inframarginal consumers remain the same, so prices are lower for any positive search cost. The hold-up motive is still present for captive buyers, and decreases in the search cost, so the direction of the comparative statics remains the same.

A distribution $G(s)$ of search costs on $s \in [\underline{s}, \overline{s}] \subset (0, 1)$ independent of the valuations yields the same results as a known s. To see this, interpret demand (1) as conditional on s and integrate it w.r.t. G to obtain the expected demand of firm i. Similarly, the firm's FOC (7), the cross-partial derivative (8) and the expression (9) in the proofs are simply integrated w.r.t. G. If the distribution of search costs is translated upward by adding $\Delta s > 0$ to each s, then the comparative statics (using the integral of (10) w.r.t. G) are the same as in Theorem 3: prices decrease in Δs. It is not surprising that if a result holds pointwise for every s in the support of the distribution G, then it holds for shifts of G. All the formulas are continuous in s, so by the Mean Value Theorem, for each result using G there exists $s \in [\underline{s}, \overline{s}]$ delivering the same result.

Suppose a fraction ν of customers are initially at neither firm and have to pay the search cost no matter which price they first learn about. Then the hold-up motive is strengthened for both firms, thus prices are higher at any $s > 0$

at which some consumers search. At large enough s, no consumers search, so each firm charges its monopoly price, which is unaffected by multiplying demand by $1-\nu$. The price decrease in the search cost becomes steeper as ν increases, because price falls from a higher level at small $s>0$ to the same monopoly level. The following proposition formalises this intuition.

Proposition 5. *For both firms, P_i and $\left|\frac{dP_i}{ds}\right|$ increase in ν.*

If no consumers know any price before search ($\nu = 1$) and all have to pay a cost for their first price observation, then the market breaks down. No matter what price the arriving consumers expect, each firm strictly prefers to charge more, for the same reason as the entrant discussed in Section 2.1.

If the firms can distinguish their initial customers from incoming switchers, then they charge the switchers a prohibitively high price. Equivalently, each firm would operate in two markets—in one as an incumbent serving its initial customers and in one as an entrant selling to the switchers. As explained at the end of Section 2, hold-up leads to market breakdown for the entrant, because no matter what price the switchers expect, the firm at which they arrive strictly prefers to charge more. Both the price for the initial customers and the price for switchers are thus independent of search cost.

Correlated valuations of consumers may change the results, depending on the joint distribution of the valuations. The only modification in the proofs is replacing $F(v_i)$ in all formulas by $F(v_i|v_j)$. The modified sufficient conditions may be harder or easier to satisfy than the original assumptions, depending on the joint distribution of the valuations. If v_X and v_Y are perfectly positively correlated, then the model with $s = 0$ is Bertrand competition, and with $s > 0$, the original Diamond (1971) paradox, where prices stay constant in s. Perfect negative correlation of v_X and v_Y reduces the environment with $s = 0$ to the Hotelling model.

An interesting case is consumers uniformly distributed on two crossing streets (on a + shape) with firm Y at the north and X at the east end of the +. In this case, both firms charge $P_i = \frac{1}{2}$ for all $s \geq 0$, so each firm's competitive and monopoly price are equal.

No outside option for the consumers (a covered market) leads to similar results as the baseline model—simply replace $\max\{0, x\}$ by x in the formulas. Then the extensive margin (more consumers entering as search costs fall) that drives the results in Moraga-González et al. (2017) and Section 6.2 of Choi et al. (2018) is absent and not needed for prices to fall in the search cost.

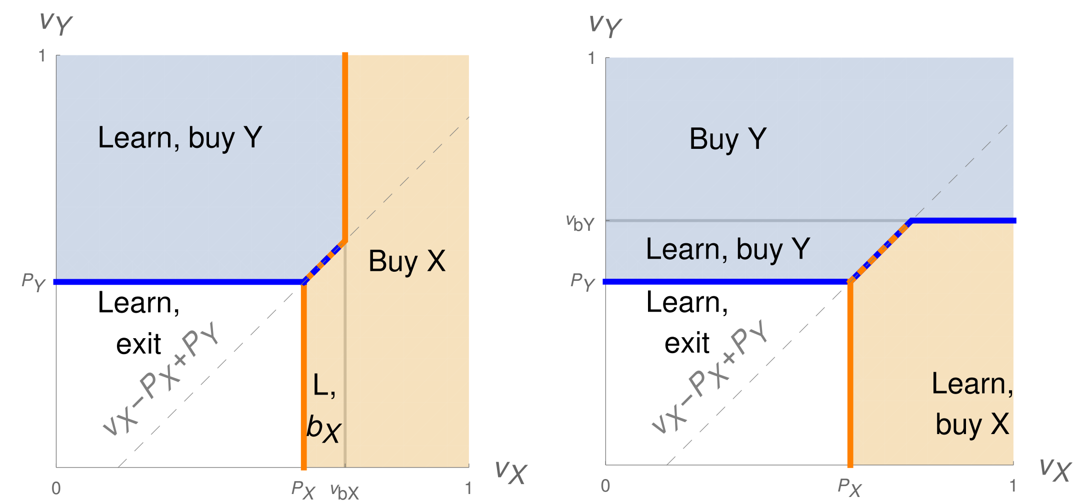
Many firms are conceptually similar to duopoly—in each firm's FOC, replace the rival firm with the combination of all rivals. The incentives of firm i are the same as when facing a single competitor which has initial demand $1-\frac{1}{n}$ and offers consumers the net value $\max_{j\neq i}\left\{v_j - P_j^*\right\}$ distributed according to $\prod_{j\neq i} F(\cdot + P_j^*)$. Equilibrium prices in an oligopoly are of course lower than in a duopoly in which all rivals are controlled by a single owner. However, the comparative statics retain their direction and increase in magnitude because the FOC of a firm with a smaller market share decreases more in s, as can be seen from (10).

If consumers initially at firm i do not know their valuation v_j for the rival firm, but can learn v_j and P_j together, then the market segmentation is depicted in Figure 3. Then valuation distributions close to uniform result in intuitive comparative statics—prices increase in the search cost.[7] However, for symmetric firms, a sufficiently fast decrease in the valuation pdf implies that prices decrease in the search cost over some range of s. A numerical example has $f_i(v_i) = \begin{cases} \frac{3}{2} & \text{if } v_i \in [0, \frac{1}{2}], \\ \frac{1}{2} & \text{if } v_i \in (\frac{1}{2}, 1], \end{cases}$ for both firms. The equilibrium prices at $s = 0$ are approximately 0.31, and the monopoly price as s becomes large is 0.25, thus lower than the duopoly price. As s increases from 0.13 to 0.19, prices decrease linearly from 0.491 to 0.384.

At large s, the environments with known and unknown valuation for the other firm are identical because no consumers switch. At $s = 0$, these models are also identical, because it is weakly dominant for all consumers to search. If consumers know their valuations, then prices jump up when the search cost becomes positive, but if the valuation for the other firm is unknown, then prices

[7] If $f_i = 1$, then the monopoly prices at large s are $P_i = \frac{1}{2}$ and the competitive prices at $s = 0$ are $P_i \approx 0.414$.

Figure 3: Demands at the pure prices $P_X = P_X^* = P_X^{\mathbb{E}} = 0.6$ and $P_Y = P_Y^* = P_Y^{\mathbb{E}} = 0.45$ and search cost $s = 0.1$. Left panel: consumers initially at firm X, right panel: Y. Orange line, including dashed: marginal consumers for X; blue line, incl. dashed: Y.



are continuous at costless search, because the mass of marginal consumers changes continuously. Thus for low positive search costs, consumers obtain greater utility when they do not know their valuation. Firms correspondingly make lower profits, so would prefer to inform consumers about their valuations.

Advertising changes the market outcome if the ad commits the firm to the advertised price. Without commitment, firms may use hidden fees to increase the actual price above the advertised level. If the price in the ad is cheap talk, then consumers do not respond to it, because each firm wants to attract as many consumers as possible by claiming to charge the lowest price. The claim is then uninformative.

By revealed preference, having unilateral commitment power cannot reduce payoff. The next proposition proves that the profit of a firm strictly increases from advertising. The intuition is that one deviation available to a firm is to advertise (and thereby commit to) its equilibrium price. The buyers make the same decision in response to a revealed price that equals what they expected in equilibrium. Because demand is the same, the profit of the deviating firm equals its equilibrium profit. Thus advertising the equilibrium price is weakly profitable. The best deviation is at least as profitable, and by revealed preference, strictly more profitable if the firm optimally changes its price.

Proposition 6. *In any equilibrium of the baseline model, giving a firm a cheap enough option to advertise its price causes it to deviate to advertising and a lower price.*

Firms choose advertising and prices simultaneously, so the rival cannot condition its price or advertising on any deviation. No punishment for deviating is possible.

Proposition 6 implies that if a firm can credibly advertise price without punishment, then it will. Then all firms advertise in equilibrium and prices are revealed to all consumers, same as if search cost was zero. The resulting profit of each firm is lower than when no firm advertises. Industry associations thus prefer to prevent members from advertising prices and to only reveal valuations to consumers.

The following section concludes with a discussion of the predictions and policy implications from the main model.

4 Discussion

When prices and profits decrease in the search cost, industry associations naturally want to provide information that helps customers compare the association members. An online directory achieves this, which justifies the cost of creating and maintaining the member database. Notably, such searchable directories do not provide price comparisons, even though these would be easy to add. A simple explanation for the lack of price information is that reducing the search cost to zero by making prices transparent would discretely decrease prices and profits compared to a small positive cost.

At low positive search costs, prices and profits are discretely higher when consumers know their valuation for each firm before the learning decision than when they learn the valuation together with the price. This is an additional motive for industry groups to inform consumers in detail about the goods and services each member provides.

For a large enough search cost, each firm is a monopolist over its initial customers, which would be a reason for high prices, especially when the exit of low-valuation buyers increases the average willingness to pay among the remaining ones. However, the exit of many consumers (who are inefficiently allocated to a firm which they value little) reduces total surplus enough to outweigh the larger share of surplus that a monopolist can obtain using its market power. Therefore firms prefer a more efficient allocation even if it means more competition.

As Adam Smith already noted, industry associations tend to collude to increase the profits of their members at the expense of consumers. A regulator maximising consumer surplus prefers either zero search cost, or if this is unattainable, then maximal cost. Prohibiting information release by an association is difficult, so the regulator should instead provide price comparisons directly. Examples already implemented are government-run health insurance exchanges, websites listing pension funds ordered by their total fee loading and public gasoline price comparison databases. Of course, the industry can counter by obfuscating prices with hidden add-on costs and private discounts. Antitrust legislation may make sharing price data among firms illegal to stop tacit collusion and prevent cartels from detecting deviations, but the tradeoff is higher prices if consumers face a search cost.

A regulator maximising total surplus unambiguously prefers a lower search cost. At small positive search costs, both kinds of regulator prefer that consumers do not know their valuation for the rival firm. However, providing price information to consumers dominates removing their valuation information even if the latter was possible.

A Proofs and results omitted from the main text

To rule out mixed pricing, some preliminaries are needed. The usual Diamond paradox proof does not work, because at any $P_i \in (0,1)$, some consumers at firm i are on the margin of leaving to firm j or exiting. Raising the price from the lower bound of mixing reduces demand, unlike in Diamond (1971).

A mixed strategy of firm i is the cdf σ_i on $[0,1]$. Fraction $\mu_i \in (0,1)$ of consumers are initially at firm i and μ_j at j. The marginal cost of firm i is c_i. The valuation v_i for firm i is distributed according to f_i with cdf F_i.

Consumer v who observes firm j's price P_j and expects firm i to choose the pricing strategy $\sigma_i^{\mathbb{E}}$ learns P_i if

$$\int_{c_i}^{1} \max\left\{0, v_i - P_i^{\mathbb{E}}, v_j - P_j\right\} d\sigma_i^{\mathbb{E}}(P_i^{\mathbb{E}}) - s \geq \max\left\{0, v_j - P_j\right\}. \tag{2}$$

The *certainty equivalent price* P_{iCE} of firm i is is the pure price of firm i that creates the same benefit of learning as $\sigma_i^{\mathbb{E}}$. The dependence of P_{iCE} on v_X, v_Y and P_j is suppressed in the notation. Formally, if $\sigma_i^{\mathbb{E}}$ puts positive probability on $P_i^{\mathbb{E}}$ s.t. $v_i - P_i^{\mathbb{E}} > \max\{0, v_j - P_j\}$, then P_{iCE} solves

$$\max\left\{v_i - P_{iCE}, 0, v_j - P_j\right\} = \int_{c_i}^{1} \max\left\{v_i - P_i^{\mathbb{E}}, 0, v_j - P_j\right\} d\sigma_i^{\mathbb{E}}(P_i^{\mathbb{E}}). \tag{3}$$

If the support of $\sigma_i^{\mathbb{E}}$ lies above $v_i - \max\{0, v_j - P_j\}$, then set $P_{iCE} := \int_{c_i}^{1} P_i^{\mathbb{E}} d\sigma_i^{\mathbb{E}}(P_i^{\mathbb{E}})$. This ensures that if $\sigma_i^{\mathbb{E}}$ is the pure P_i^*, then $P_{iCE} = P_i^*$ for any level of P_i^*.

The demand for a firm consists of consumers initially at that firm who either buy immediately or learn and then buy from that firm, and consumers initially at the rival firm who learn and switch. Customers initially at i buy immediately from i if $v_i - P_i \geq \max\{0, v_j - P_{jCE} - s\}$. They learn and then buy from i when $v_j - P_{jCE} - s \geq \max\{0, v_i - P_i\}$ and $v_i - P_i \geq \max\{0, v_j - P_j\}$. These conditions can be combined to $v_i \geq P_i + \max\{0, v_j - \max\{P_{jCE} + s, P_j\}\}$.

Consumers starting at j buy from i if both their expected payoff from learning $v_i - P_{iCE} - s$ and the observed payoff $v_i - P_i$ from i after learning are larger than the payoff $\max\{0, v_j - P_j\}$ from exiting or buying from j. Combining these conditions results in $v_i \geq \max\{P_i, P_{iCE} + s\} + \max\{0, v_j - P_j\}$.

The demand that firm i expects from price P_i when it expects firm j to choose pricing strategy σ_j^*, consumers initially at j to expect $\sigma_i^{\mathbb{E}}$ with certainty equivalent price P_{iCE} and consumers at i to expect $\sigma_j^{\mathbb{E}}$ with P_{jCE} is

$$D_i(P_i, \sigma_j^*, \sigma_i^{\mathbb{E}}, \sigma_j^{\mathbb{E}}) = \mu_i \int_{c_j}^1 \int_0^1 \int_{P_i + \max\left\{0, v_j - \max\left\{P_{jCE}+s, P_j^*\right\}\right\}}^1 f_i(v_i) f_j(v_j) dv_i dv_j d\sigma_j^*(P_j^*)$$
$$+ \mu_j \int_{c_j}^1 \int_0^1 \int_{\max\{P_i, P_{iCE}+s\} + \max\left\{0, v_j - P_j^*\right\}}^1 f_i(v_i) f_j(v_j) dv_i dv_j d\sigma_j^*(P_j^*). \quad (4)$$

The outer integral in (4) reflects firm i's expectation over the prices of firm j.

Having derived the demand, the next lemma establishes pure best responses of the firms. A sufficient condition is that the densities of consumer valuations do not decrease too fast. The uniform and truncated exponential distributions satisfy the condition, as does any increasing f_i and any f_i that decreases slower than $\exp(-v_i^2/4)$.

Lemma 7. *If $(P_i - c_i)\frac{\partial f_i(P_i+w)}{\partial P_i} \geq -2 f_i(P_i + w)$ for all $P_i \in (c_i, 1)$ and $w \in [0, 1 - \max\{P_i, c_j\})$ for each firm i, then each has a pure best response to any $\sigma_j^*, \sigma_i^{\mathbb{E}}, \sigma_j^{\mathbb{E}}$.*

Proof of Lemma 7. Using (4), firm i's marginal profit is

$$\begin{aligned}\frac{\partial \pi_i(P_i, \sigma_j^*, \sigma_i^{\mathbb{E}}, \sigma_j^{\mathbb{E}})}{\partial P_i} = & \int_{c_j}^1 \int_0^1 \left[1 - \mu_i F_i\left(P_i + \max\left\{0, v_j - \max\left\{P_{jCE} + s, P_j^*\right\}\right\}\right)\right. \\ & - \mu_j F_i\left(\max\{P_i, P_{iCE} + s\} + \max\left\{0, v_j - P_j^*\right\}\right) \quad (5) \\ & - (P_i - c_i)\mu_i f_i\left(P_i + \max\left\{0, v_j - \max\left\{P_{jCE} + s, P_j^*\right\}\right\}\right) \\ & \left. -(P_i - c_i)\mu_j \mathbf{1}_{\{P_i > P_{iCE}+s\}} f_i\left(P_i + \max\left\{0, v_j - P_j^*\right\}\right)\right] dF_j(v_j) d\sigma_j^*(P_j^*).\end{aligned}$$

and the second derivative

$$
\begin{aligned}
\frac{\partial^2 \pi_i}{\partial P_i^2} = &- \int_{c_j}^{1} \int_0^1 \Bigg[2\mu_i f_i \left(P_i + \max\left\{0, v_j - \max\left\{P_{jCE} + s, P_j^*\right\}\right\}\right) \qquad (6) \\
&+ 2\mu_j \mathbf{1}_{\{P_i > P_{iCE}+s\}} f_i \left(P_i + \max\left\{0, v_j - P_j^*\right\}\right) \\
&+ (P_i - c_i)\mu_i \frac{\partial f_i \left(P_i + \max\left\{0, v_j - \max\left\{P_{jCE} + s, P_j^*\right\}\right\}\right)}{\partial P_i} \\
&+ (P_i - c_i)\mu_j \mathbf{1}_{\{P_i > P_{iCE}+s\}} \frac{\partial f_i \left(P_i + \max\left\{0, v_j - P_j^*\right\}\right)}{\partial P_i} \Bigg] dF_j(v_j) d\sigma_j^*(P_j^*).
\end{aligned}
$$

Sufficient for $\frac{\partial^2 \pi_i}{\partial P_i^2} < 0 \; \forall P_i \in (c_i, 1)$ is $\int_0^1 [(P_i - c_i)\frac{\partial f_i(P_i+w)}{\partial P_i} + 2f_i(P_i+w)] dF_j(v_j) \geq 0$ for all $w \in [0, 1-c_j)$ and $P_i \in (c_i, 1)$. This is ensured if $(P_i - c_i)\frac{\partial f_i(P_i+w)}{\partial P_i} \geq -2f_i(P_i+w)$ for all $P_i \in (c_i, 1)$ and $w \in [0, 1-c_j)$ and $P_i + w \in [0, 1)$. Then the best response (BR) of firm i to any $\sigma_j^*, \sigma_i^{\mathbb{E}}, \sigma_j^{\mathbb{E}}$ is pure and unique. $\square$

The conditions in Lemma 7 are far from necessary for pure equilibria. For example, if just one firm has a unique pure best response to any undominated strategy of the competitor, then generically the equilibrium is pure.

Proof of Lemma 1. Using (1), firm i's expected profit has the derivative

$$
\begin{aligned}
\frac{\partial \pi_i(P_i, P_j^*, P_i^{\mathbb{E}}, P_j^*)}{\partial P_i} = &\frac{1}{2} \int_0^1 \left[1 - F\left(P_i + \max\left\{0, v_j - P_j^* - s\right\}\right)\right. \qquad (7) \\
&+ 1 - F\left(\max\left\{P_i, P_i^{\mathbb{E}} + s\right\} + \max\left\{0, v_j - P_j^*\right\}\right) - P_i f\left(P_i + \max\left\{0, v_j - P_j^* - s\right\}\right) \\
&\left. - P_i \mathbf{1}_{\left\{P_i > P_i^{\mathbb{E}}+s\right\}} f\left(P_i + \max\left\{0, v_j - P_j^*\right\}\right)\right] dF(v_j).
\end{aligned}
$$

By Milgrom and Roberts (1990) Theorem 4, the game is supermodular if $\frac{\partial^2 \pi_i}{\partial P_i \partial P_j^*} \geq 0$, in which case prices are strategic complements. The cross-partial derivative is

$$\frac{\partial^2 \pi_i}{\partial P_i \partial P_j^*} = \frac{1}{2}\int_{P_j^*+s}^{1}\left[f\left(P_i+v_j-P_j^*-s\right)+P_i\frac{\partial f\left(P_i+v_j-P_j^*-s\right)}{\partial P_i}\right]dF(v_j) \tag{8}$$

$$+\frac{1}{2}\int_{P_j^*}^{1}\left[f\left(\max\left\{P_i,P_i^{\mathbb{E}}+s\right\}+v_j-P_j^*\right)+P_i\mathbf{1}_{\left\{P_i>P_i^{\mathbb{E}}+s\right\}}\frac{\partial f\left(P_i+v_j-P_j^*\right)}{\partial P_i}\right]dF(v_j)$$

because $\frac{\partial f}{\partial P_j^*} = -\frac{\partial f}{\partial P_i}$. Sufficient for $\frac{\partial^2\pi_i}{\partial P_i \partial P_j^*} > 0$ is $\int_0^1 [P_i \frac{\partial f(P_i+w)}{\partial P_i} + f(P_i + w)]dF(v_j) \geq 0$ for all $P_i \in (0,1)$ and $P_i + w \in [0,1)$, which is ensured if $P_i \frac{\partial f(P_i+w)}{\partial P_i} \geq -f(P_i+w)$ for all $P_i \in (0,1)$ and $w \in [0, 1-P_i)$. □

Proof of Lemma 2. Profit after imposing the equilibrium condition $P_i = P_i^{\mathbb{E}} = P_i^*$ on both firms is denoted π_i^*. Sufficient for uniqueness is that the slopes of best responses are below 1, i.e., $\left|-\frac{\partial^2\pi_i^*}{\partial P_i \partial P_j} \middle/ \frac{\partial^2 \pi_i^*}{\partial P_i^2}\right| < 1$ for each firm $i \neq j$. Equivalently, $\frac{\partial^2\pi_i^*}{\partial P_i \partial P_j} + \frac{\partial^2\pi_i^*}{\partial P_i^2} < 0$.

Use $\mathbf{1}_{\left\{P_i>P_i^{\mathbb{E}}+s\right\}} = 0$ in (7) and (8) to obtain

$$\frac{\partial^2\pi_i^*}{\partial P_i \partial P_j} + \frac{\partial^2\pi_i^*}{\partial P_i^2} = \frac{1}{2}\int_{P_j+s}^{1} f\left(P_i+v_j-P_j-s\right)dF(v_j) + \frac{1}{2}\int_{P_j}^{1} f\left(P_i+v_j-P_j+s\right)dF(v_j)$$
$$+\frac{1}{2}\int_{P_j+s}^{1} P_i\frac{\partial f\left(P_i+v_j-P_j-s\right)}{\partial P_i}dF(v_j) + \frac{1}{2}\int_0^1 -f\left(P_i+\max\left\{0,v_j-P_j-s\right\}\right)dF(v_j)$$
$$+\frac{1}{2}\int_0^1\left[-f\left(P_i+\max\left\{0,v_j-P_j-s\right\}\right)-P_i\frac{\partial f\left(P_i+\max\left\{0,v_j-P_j-s\right\}\right)}{\partial P_i}\right]dF(v_j),$$

which equals $-\frac{1}{2}\int_0^{P_j+s} f\left(P_i\right)dF(v_j) + \frac{1}{2}\int_{P_j}^1 f\left(P_i+v_j-P_j+s\right)dF(v_j) - \frac{1}{2}\int_0^{P_j+s} P_i \frac{\partial f(P_i)}{\partial P_i} dF(v_j) - \frac{1}{2}\int_0^1 f\left(P_i+\max\left\{0,v_j-P_j-s\right\}\right)dF(v_j)$, which further simplifies to

$$-\frac{1}{2}\left[2f\left(P_i\right)+P_i\frac{\partial f\left(P_i\right)}{\partial P_i}\right]F(P_j+s) + \frac{1}{2}\int_{P_j}^1\left[f\left(P_i+v_j-P_j+s\right)-f\left(P_i+v_j-P_j-s\right)\right]dF(v_j).$$

Sufficient for uniqueness are $P_i\frac{\partial f(P_i)}{\partial P_i} \geq -2f(P_i)$ and $f'(x) \leq 0 \; \forall x$. □

Proof of Theorem 3. The FOC of firm i in (7) after imposing the equilibrium condition $P_i = P_i^{\mathbb{E}} = P_i^*$ for each firm is

$$FOC_i^* = \frac{1}{2}\int_0^1 \left[1 - F\left(P_i + \max\{0, v_j - P_j - s\}\right)\right. \tag{9}$$
$$\left. + 1 - F\left(P_i + s + \max\{0, v_j - P_j\}\right) - P_i f\left(P_i + \max\{0, v_j - P_j - s\}\right)\right] dF(v_j).$$

Its derivative w.r.t. s is

$$\frac{\partial FOC_i^*}{\partial s} = \frac{1}{2}\int_{P_j+s}^1 \left[f\left(P_i + v_j - P_j - s\right) + P_i \frac{\partial f\left(P_i + v_j - P_j - s\right)}{\partial P_i}\right] dF(v_j)$$
$$- \frac{1}{2}\int_0^1 f\left(P_i + s + \max\{0, v_j - P_j\}\right) dF(v_j), \tag{10}$$

negative if $f\left(P_i - s + w\right) + P_i \frac{\partial f(P_i - s + w)}{\partial P_i} \leq f\left(P_i + s + w\right)$ for all $P_i - s + w \in (0,1)$, $w \geq 0$. For specific distributions, (10) can be calculated explicitly. Sufficient for $\frac{\partial FOC_i^*}{\partial s} < 0$ is that f is uniform or f is truncated exponential and $(1 - P_i)\exp\left(-P_i - 1 + P_j + s\right) \leq \exp\left(-P_i - 1 + P_j - s\right)$.

By the Implicit Function Theorem, $\begin{bmatrix} \frac{dP_X^*}{ds} \\ \frac{dP_Y^*}{ds} \end{bmatrix} = -\begin{bmatrix} \frac{\partial FOC_X^*}{\partial P_X} & \frac{\partial FOC_X^*}{\partial P_Y} \\ \frac{\partial FOC_Y^*}{\partial P_X} & \frac{\partial FOC_Y^*}{\partial P_Y} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial FOC_X^*}{\partial s} \\ \frac{\partial FOC_Y^*}{\partial s} \end{bmatrix} = -\frac{1}{det(D_P FOC^*)} \begin{bmatrix} \frac{\partial FOC_Y^*}{\partial P_Y} & -\frac{\partial FOC_X^*}{\partial P_Y} \\ -\frac{\partial FOC_Y^*}{\partial P_X} & \frac{\partial FOC_X^*}{\partial P_X} \end{bmatrix} \begin{bmatrix} \frac{\partial FOC_X^*}{\partial s} \\ \frac{\partial FOC_Y^*}{\partial s} \end{bmatrix}$. The matrix $\left[\frac{\partial FOC_i^*}{\partial P_j}\right]^{-1}$ is negative semidefinite iff the equilibrium is stable. Therefore sufficient for $\frac{dP_i^*}{ds} < 0$ in any stable equilibrium is $\frac{\partial FOC_i^*}{\partial s} < 0$.

If no consumers learn, then s does not affect prices, so $\frac{\partial FOC_i^*}{\partial s} \leq 0$. The condition $1 - P_i^* - s > -P_j^*$ ensures that some consumers at j learn about i, given the equilibrium prices. In a symmetric equilibrium $P_i^* = P_j^*$, so some consumers at both firms learn if $s < 1$. □

Proof of Prop. 4. Let $\rho_k := \frac{dP_k}{ds}$. Write the equilibrium condition FOC_i^* (9) of firm i in terms of a change δ in the search cost from some baseline (e.g., initial equilibrium before a change in parameters) and a change $\rho_k\delta$ in P_k as

$$
\begin{aligned}
&FOC_i^* = \frac{1}{2}\int_0^{P_j+\rho_j\delta+s+\delta} \left[1 - F\left(P_i + \rho_i\delta\right)\right] dF(v_j) \qquad (11)\\
&+ \frac{1}{2}\int_{P_j+\rho_j\delta+s+\delta}^{1} \left[1 - F\left(P_i + \rho_i\delta + v_j - P_j - \rho_j\delta - s - \delta\right)\right] dF(v_j)\\
&+ \frac{1}{2}\int_0^{P_j+\rho_j\delta} \left[1 - F\left(P_i + \rho_i\delta + s + \delta\right)\right] dF(v_j)\\
&+ \frac{1}{2}\int_{P_j+\rho_j\delta}^{1} \left[1 - F\left(P_i + \rho_i\delta + s + \delta + v_j - P_j - \rho_j\delta\right)\right] dF(v_j)\\
&- \frac{1}{2}\int_0^{P_j+\rho_j\delta+s+\delta} (P_i + \rho_i\delta) f\left(P_i + \rho_i\delta\right) dF(v_j)\\
&- \frac{1}{2}\int_{P_j+\rho_j\delta+s+\delta}^{1} (P_i + \rho_i\delta) f\left(P_i + \rho_i\delta + v_j - P_j - \rho_j\delta - s - \delta\right) dF(v_j).
\end{aligned}
$$

The derivative of FOC* w.r.t. δ is (because the derivatives w.r.t. the bounds of the integrals cancel)

$$
\begin{aligned}
&- \rho_i\frac{1}{2}\int_0^{P_j+\rho_j\delta+s+\delta} f\left(P_i + \rho_i\delta\right) dF(v_j) \qquad (12)\\
&- (\rho_i - \rho_j - 1)\frac{1}{2}\int_{P_j+\rho_j\delta+s+\delta}^{1} f\left(P_i + \rho_i\delta + v_j - P_j - \rho_j\delta - s - \delta\right) dF(v_j)\\
&- (\rho_i + 1)\frac{1}{2}\int_0^{P_j+\rho_j\delta} f\left(P_i + \rho_i\delta + s + \delta\right) dF(v_j)\\
&- (\rho_i - \rho_j + 1)\frac{1}{2}\int_{P_j+\rho_j\delta}^{1} f\left(P_i + \rho_i\delta + s + \delta + v_j - P_j - \rho_j\delta\right) dF(v_j)\\
&- \rho_i\frac{1}{2}\int_0^{P_j+\rho_j\delta+s+\delta} \left[f\left(P_i + \rho_i\delta\right) + (P_i + \rho_i\delta)\frac{\partial f\left(P_i + \rho_i\delta\right)}{\partial P_i}\right] dF(v_j)\\
&- \frac{1}{2}\int_{P_j+\rho_j\delta+s+\delta}^{1} [\rho_i f\left(P_i + \rho_i\delta + v_j - P_j - \rho_j\delta - s - \delta\right)\\
&+ (\rho_i - \rho_j - 1)(P_i + \rho\delta)\frac{\partial f\left(P_i + \rho_i\delta + v_j - P_j - \rho_j\delta - s - \delta\right)}{\partial P_i}] dF(v_j).
\end{aligned}
$$

Equilibrium implies $\frac{dFOC_i^*}{d\delta} = 0$ for both firms because $FOC_i^* = 0$ at any s. The first four lines of (11) are D_i and the first four lines of (12) are $\frac{dD_i}{d\delta}$.

If $f(P_i) + P_i \frac{\partial f(P_i)}{\partial P_i} \geq 0$ for all P_i and $|\rho_i - \rho_j| < 1$ (e.g., in a symmetric equilibrium), then the last three lines of (12) are positive, because $\rho_k = \frac{dP_k}{ds} < 0$ by Theorem 3. Then $\frac{dD_i}{d\delta} < 0$ for both firms and the mass of exiting consumers increases in the search cost.

The total surplus TS is

$$TS := \mu_i \iint_{v_i - P_i \geq \max\left\{0, v_j - \max\left\{P_j^{\mathbb{E}} + s, P_j^*\right\}\right\}} (v_i - c_i) dF_i(v_i) dF_j(v_j) \tag{13}$$
$$+ \mu_j \iint_{v_i - \max\left\{P_i, P_i^{\mathbb{E}} + s\right\} > \max\left\{0, v_j - P_j^*\right\}} (v_i - c_i) dF_i(v_i) dF_j(v_j)$$
$$+ \mu_j \iint_{v_j - P_j \geq \max\left\{0, v_i - \max\left\{P_i^{\mathbb{E}} + s, P_i^*\right\}\right\}} (v_j - c_j) dF_j(v_j) dF_i(v_i)$$
$$+ \mu_i \iint_{v_j - \max\left\{P_j, P_j^{\mathbb{E}} + s\right\} > \max\left\{0, v_i - P_i^*\right\}} (v_j - c_j) dF_j(v_j) dF_i(v_i)$$

If $\frac{dD_i}{d\delta} < 0$ for both firms, then $\frac{dTS}{d\delta} < 0$ because the integration region in TS is the same as in $D_i + D_j$ and on it, the integrated functions $v_i - c_i$, $v_j - c_j$ are nonnegative and do not depend on δ. Profit is $P_i D_i$. Because $\frac{dP_i}{ds} < 0$ and $\frac{dD_i}{ds} = \frac{dD_i}{d\delta} < 0$, profit falls in s.

Consumer surplus CS just replaces c_i with P_i in (13). Because prices fall in the search cost, $\frac{dCS}{d\delta} \gtreqless 0$. □

Proof of Prop. 5. Compared to (1), the mass of consumers initially at each firm is multiplied by $1-\nu$. The mass ν of unattached customers with $v_i - P_i^{\mathbb{E}} > v_j - P_j^{\mathbb{E}}$ learn firm i's price if $v_i - P_i^{\mathbb{E}} \geq s$ and then buy from i if both $v_i \geq P_i$ and $v_i - P_i \geq v_j - \max\left\{P_j^{\mathbb{E}} + s, P_j^*\right\}$. Demand is

$$\frac{1-\nu}{2} \int_0^1 \int_{P_i + \max\left\{0, v_j - \max\left\{P_j^{\mathbb{E}} + s, P_j^*\right\}\right\}}^1 f(v_i) f(v_j) dv_i dv_j$$
$$+ \frac{1-\nu}{2} \int_0^1 \int_{\max\left\{P_i, P_i^{\mathbb{E}} + s\right\} + \max\left\{0, v_j - P_j^*\right\}}^1 f(v_i) f(v_j) dv_i dv_j$$
$$+ \nu \int_0^1 \int_{\max\left\{P_i^{\mathbb{E}} + v_j - P_j^{\mathbb{E}},\ P_i^{\mathbb{E}} + s,\ P_i,\ P_i + v_j - \max\left\{P_j^{\mathbb{E}} + s, P_j^*\right\}\right\}}^1 f(v_i) f(v_j) dv_i dv_j.$$

The FOC after imposing the equilibrium condition is

$$FOC_i^* = \frac{1-\nu}{2}\int_0^1 \left[1 - F\left(P_i + \max\{0, v_j - P_j - s\}\right)\right] dF(v_j)$$
$$+\frac{1-\nu}{2}\int_0^1 \left[1 - F\left(P_i + s + \max\{0, v_j - P_j\}\right) - P_i f\left(P_i + \max\{0, v_j - P_j - s\}\right)\right] dF(v_j)$$
$$+\nu\int_0^1 \left[1 - F\left(P_i + \max\{s, v_j - P_j\}\right)\right] dF(v_j) = 0.$$

Compared to (9) in which $\nu = 0$, the derivative $\frac{\partial FOC_i^*}{\partial s} < 0$ is larger in magnitude and the matrix $\left[\frac{\partial FOC_i^*}{\partial P_j}\right]$ the same. The Implicit Function Theorem then yields $\frac{dP_i}{ds} < 0$ larger in magnitude. □

Proof of Prop. 6. Denote the profit of firm i from advertising and committing to price P_i by $\pi_i^a(P_i, P_j^*, P_j^{\mathbb{E}})$. Denote by $\pi_i(P_i, P_j^*, P_i^{\mathbb{E}}, P_j^{\mathbb{E}})$ the no-advertising profit. For firm i who advertises its price when its rival j does not, demand is

$$\begin{aligned} D_i^a(P_i, P_j^*, P_j^{\mathbb{E}}) = &\frac{1}{2}\int_0^1\int_{P_i+\max\left\{0, v_j - P_j^{\mathbb{E}} - s\right\}}^1 dF_i(v_i) dF_j(v_j) \\ &+ \frac{1}{2}\int_0^1\int_{P_i+s+\max\left\{0, v_j - P_j^*\right\}}^1 dF_i(v_i) dF_j(v_j). \end{aligned} \tag{14}$$

Demand D_i from not advertising only replaces the last P_i with $P_i^{\mathbb{E}}$. The profit of firm i from advertising its equilibrium price $P_i^* = P_i^{\mathbb{E}}$ equals its no-ad equilibrium profit.

At the price P_i^* that maximises the no-ad profit, the FOC $\frac{\partial \pi_i(P_i)}{\partial P_i} = 0$ holds. The FOC for a unilaterally advertising firm is

$$\begin{aligned} \frac{\partial \pi_i^a(P_i)}{\partial P_i} = &\int_0^1 \left[\frac{1}{2}F\left(P_i + \max\left\{0, v_j - P_j^* - s\right\}\right)\right. \\ &+ \frac{1}{2}F\left(P_i + s + \max\left\{0, v_j - P_j^*\right\}\right) - P_i\frac{1}{2}f\left(P_i + \max\left\{0, v_j - P_j^* - s\right\}\right) \\ &\left. - P_i\frac{1}{2}f\left(P_i + s + \max\left\{0, v_j - P_j^*\right\}\right)\right] dF(v_j). \end{aligned} \tag{15}$$

The only change in the FOC compared to unadvertised P_i is that the last line of (15) is negative instead of zero. The marginal profit at any P_i, including P_i^*, is thus lower for a firm deviating to advertising: $\frac{\partial \pi_i^a(P_i^*, P_j^*, P_j^{\mathbb{E}})}{\partial P_i} < 0 = \frac{\partial \pi_i(P_i^*, P_j^*, P_i^{\mathbb{E}}, P_j^{\mathbb{E}})}{\partial P_i}$. This together with $\pi_i^a(P_i^*, P_j^*, P_j^{\mathbb{E}}) = \pi_i(P_i^*, P_j^*, P_i^{\mathbb{E}}, P_j^{\mathbb{E}})$ implies that π_i^a crosses π_i from above at P_i^*. So for any $\epsilon > 0$ small enough, $\pi_i^a(P_i^* - \epsilon, P_j^*, P_j^{\mathbb{E}}) > \pi_i(P_i^* - \epsilon, P_j^*, P_i^{\mathbb{E}}, P_j^{\mathbb{E}})$. If the cost of advertising is smaller than $\pi_i^a(P_i^* - \epsilon, P_j^*, P_j^{\mathbb{E}}) - \pi_i(P_i^* - \epsilon, P_j^*, P_i^{\mathbb{E}}, P_j^{\mathbb{E}})$, then firm i strictly prefers to advertise and cut price. □

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